EXAM C QUESTIONS OF THE WEEK

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Question 7 - Week of September 5

An insurer is analyzing a new rating system in which auto policyholders are classified as either low risk or high risk. The insurer has the following data for time (in years) until first claim for 5 policyholders in each classification.

Low Risk: 2 , 4 , 7 , 7 , 10+ (+ indicates a censored observation).

High Risk: 1 , 3 , 3 , 6 , 7

The insurer uses the Cox proportional hazards model with low risk as the baseline. A single covariate Z is used to distinguish low risk ($Z = 0$) and high risk ($Z = 1$).

(a) Write out the expression for the partial likelihood function.

(b) The maximum likelihood estimate of β is found to be .7467 .
Use the Breslow estimate of $H_0(t)$ to estimate the probability that the first claim for a low risk policyholder occurs within 5 years. Calculate the same probability for a high risk policyholder.

The solution can be found below.

Question 7 Solution

(a) At each "death" point we have factor which has a numerator of 1 of the death is low risk, and e^{β} for each death that is high risk. The denominator of the factor at a death point is the sum of the number of low risk still at risk and e^{β} times the number of high risk still at risk.

The partial likelihood function is

$$\frac{e^{\beta}}{5+5e^{\beta}} \times \frac{1}{5+4e^{\beta}} \times \left(\frac{e^{\beta}}{4+4e^{\beta}}\right)^2 \times \frac{1}{4+2e^{\beta}} \times \frac{e^{\beta}}{3+2e^{\beta}} \times \frac{e^{\beta}}{(3+e^{\beta})^3} .$$

(b) The Breslow estimate of $H_0(5)$ is

$$\widehat{H}_0(5) = \frac{1}{5+5e^{\beta}} + \frac{1}{5+4e^{\beta}} + \frac{2}{4+4e^{\beta}} + \frac{1}{4+2e^{\beta}} = .421 .$$

The estimate of $S_0(5)$ is $e^{-\widehat{H}_0(5)} = .656$. This is the probability that a low risk driver's first claim is after time 5, so the probability in question is .344 .

The probability that a high risk driver's first claim is after time 5 is $S_1(5) = [S_0(5)]^{e^{\beta}}$, so the estimate is $(.656)^{e^{.7467}} = .411$, and the estimate of the probability that the first claim is before time 5 is .589.