EXAM M QUESTIONS OF THE WEEK

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Week of September 4/06

(x) and (y) are lives subject to a common shock mortality model, with common shock parameter λ.

(x) has a constant force of mortality of 2λ, and (y) has a constant force of mortality of 2.5λ.

Find ${}_{\infty}q_{\overset{1}{x}y}$ and ${}_{\infty}q_{x\overset{1}{y}}$.

Interpret verbally the quantity $1 - ({}_{\infty}q_{\overset{1}{x}y} + {}_{\infty}q_{x\overset{1}{y}})$.

The solution can be found below.

Week of September 4/06 - Solution

${}_{\infty}q_{\overset{1}{x}y}$ is defined as $P(T(x) < T(y)) = \int_0^{\infty} {}_tp_{xy}\,\mu_x(t)\,dt$, however this definition applies only if there is no common shock. When there is a common shock, the definition is $\int_0^{\infty} {}_tp_{xy}\,\mu_x^*(t)\,dt$, where $\mu_x^*(t)$ is the force of mortality for (x) excluding the common shock component.

$\mu_x(t) = \mu_x^*(t) + \lambda \rightarrow \mu_x^*(t) = \lambda$,
and in a similar way, $\mu_y^*(t) = 1.5\lambda$.

Then, $\mu_{xy}(t) = \mu_x^*(t) + \mu_y^*(t) + \lambda = 3.5\lambda$, and ${}_tp_{xy} = e^{-3.5\lambda}$.

${}_{\infty}q_{\overset{1}{x}y} = \int_0^{\infty} {}_tp_{xy}\,\mu_x^*(t)\,dt = \int_0^{\infty} e^{-3.5\lambda} \cdot \lambda\,dt = \frac{\lambda}{3.5\lambda} = \frac{1}{3.5}$,
and
${}_{\infty}q_{x\overset{1}{y}} = \int_0^{\infty} {}_tp_{xy}\,\mu_y^*(t)\,dt = \int_0^{\infty} e^{-3.5\lambda} \cdot \lambda\,dt = \frac{1.5\lambda}{3.5\lambda} = \frac{1.5}{3.5}$.

$1 - ({}_{\infty}q_{\overset{1}{x}y} + {}_{\infty}q_{x\overset{1}{y}}) = 1 - (\frac{1}{3.5} + \frac{1.5}{3.5}) = \frac{1}{3.5}$.
This is the probability
$1 - [P(T(x) < T(y)) + P(T(y) < T(x))] = P(T(x) = T(y))$
(that neither (x) nor (y) is the first to die).
Therefore, it is the probability that they die simultaneously by common shock.
This can also be formulated in general is the form $\int_0^{\infty} {}_tp_{xy}\,\lambda\,dt$,
which becomes $\int_0^{\infty} e^{-3.5\lambda} \cdot \lambda\,dt = \frac{1}{3.5}$ in this case.