

## EXAM FM QUESTIONS OF THE WEEK

S. Broverman, 2005

### **Question 10 - Week of September 26**

You are given the following term structure of annual effective yield rates for zero-coupon bonds of different maturities.

Maturity (in years)	Yield
1	.06
2	$i$
3	.08

We wish to find the complete range of values of  $i$  that result in the following two conditions being satisfied:

- (i) the one year forward, one year effective interest rate (i.e., for the 2nd year) is at least 7%, and
- (ii) the two year forward, one year effective interest rate (i.e., for the 3rd year) is at least 7%.

The solution can be found below.

### **Question 10 Solution**

We denote the one year forward, one year effective rate of interest by  $f$ , and the two year forward, one year effective rate of interest by  $g$ .

The relationship between  $f$  and the zero-coupon yields is  $(1.06)(1 + f) = (1 + i)^2$ , and the relationship between  $g$  and the zero-coupon yields is  $(1 + i)^2(1 + g) = (1.08)^3$ .

In order to have  $f > .07$ , we must have  $(1 + i)^2 > (1.06)(1.07)$ , so that  $i > .065$ .

In order to have  $g > .07$ , we must have  $\frac{1}{1+g} < \frac{1}{1.07}$ , so that  $(1 + i)^2 = \frac{(1.08)^3}{1+g} = 1.1773$ , and  $i < .085$ .

Therefore, the range for  $i$  is  $.065 < i < .085$ .

Note also that since  $(1.06)(1 + f)(1 + g) = (1.08)^3$ , in order for  $f > .07$ , we must have  $1 + g = \frac{(1.08)^3}{(1.06)(1+f)} < 1.1107$ , and similarly for  $f$ .