

EXAM FM QUESTIONS OF THE WEEK

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Week of September 25/06

You are given the following bond prices for bonds with annual coupons and face and redemption amounts of \$100 each.

Coupon Rate	Term to Maturity	Price
6%	2 yrs.	97.36
6%	3 yrs.	94.97
10%	3 yrs.	105.36

Find the annual effective rates of interest for a one, two and three-year zero-coupon bonds.

The solution can be found below.

Week of September 25/06 - Solution

We will denote by s_1 , s_2 and s_3 the one, two and three-year spot rates.

$$\text{Then } 97.36 = \frac{6}{1+s_1} + \frac{106}{(1+s_2)^2} ,$$

$$\text{and } 94.97 = \frac{6}{1+s_1} + \frac{6}{(1+s_2)^2} + \frac{106}{(1+s_3)^3}$$

$$\text{and } 105.36 = \frac{10}{1+s_1} + \frac{10}{(1+s_2)^2} + \frac{110}{(1+s_3)^3} .$$

$$\text{Then, } 10(94.97) - 6(105.36) = 317.54$$

$$= 10\left[\frac{6}{1+s_1} + \frac{6}{(1+s_2)^2} + \frac{106}{(1+s_3)^3}\right] - 6\left[\frac{10}{1+s_1} + \frac{10}{(1+s_2)^2} + \frac{110}{(1+s_3)^3}\right] = \frac{400}{(1+s_3)^3} .$$

$$\text{It follows that } \frac{1}{(1+s_3)^3} = \frac{317.54}{400} = .79385 , \text{ and } s_3 = \left(\frac{400}{317.54}\right)^{1/3} - 1 = .080 .$$

$$\text{Then } 94.97 = \frac{6}{1+s_1} + \frac{6}{(1+s_2)^2} + 106(.79385) ,$$

$$\text{from which it follows that } \frac{6}{1+s_1} + \frac{6}{(1+s_2)^2} = 10.82 .$$

Subtracting this from the first bond gives us

$$97.36 - 10.82 = 86.54 = \frac{6}{1+s_1} + \frac{106}{(1+s_2)^2} - \left[\frac{6}{1+s_1} + \frac{6}{(1+s_2)^2}\right] = \frac{100}{(1+s_2)^3}$$

$$\text{from which it follows that } s_2 = \left(\frac{100}{86.54}\right)^{1/2} - 1 = .075 .$$

$$\text{Using the first bond, we have } 97.36 = \frac{6}{1+s_1} + \frac{106}{(1+s_2)^2} = \frac{6}{1+s_1} + \frac{106}{(1.075)^2} ,$$

$$\text{from which it follows that } 5.63 = \frac{6}{1+s_1} \text{ and } s_1 = \frac{6}{5.63} - 1 = .066 .$$