EXAM C QUESTIONS OF THE WEEK

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A Bayesian model has a model distribution X which is negative binomial with parameters r and $\beta = 1$. The parameter r has a prior distribution which is exponential with a mean of 1.

A single sample value of X is observed to be $X = 1$.

Find the posterior density of r.

Solution can be found below.

Week of September 25/06 - Solution

The posterior density of r is $\pi(r|X=1) = \frac{f(1,r)}{P(X=1)}$,
where $f(x,r)$ is the joint density of X and r, and $P(X=1)$ is the marginal probability of X.

$f(1,r) = P(X=1|r) \cdot \pi(r)$, where $\pi(r)$ is the prior density of r.

We are given that $\pi(r) = e^{-r}$ (exponential with a mean of 1).

We are also given that $P(X=1|r) = \frac{r}{2^{r+1}}$ (negative binomial with $\beta = 1$) .

The marginal probability for X is found from

$$P(X=1) = \int_0^\infty f(1,r)\,dr = \int_0^\infty P(X=1|r) \cdot \pi(r)\,dr = \int_0^\infty \frac{r}{2^{r+1}} \cdot e^{-r}\,dr = \frac{1}{2}\int_0^\infty r \cdot (2e)^{-r}\,dr$$

We use the integration rule $\int_0^\infty te^{-at}\,dt = \frac{1}{a^2}$, with $a = ln(2e)$, so that
$P(X=1) = \frac{1}{2} \cdot \frac{1}{[ln(2e)]^2} = .2953$.

The posterior density is $\pi(r|X=1) = \frac{f(1,r)}{P(X=1)} = [\frac{r}{2^{r+1}} \cdot e^{-r}] \Big/ .2953$.