EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 9 - Week of September 19

X and Y are normal random variables with means μ_X and μ_Y, standard deviations σ_X and σ_Y, and correlation coefficient ρ. $F_X(t)$ and $F_Y(t)$ denote the cdf's of X and Y respectively.
If $\sigma_X = 2\sigma_Y$, for what values of t is it true that $F_X(t) \geq F_Y(t)$?

A) $t \leq 2\mu_Y - \mu_X$ B) $t \leq 2\mu_X - \mu_Y$ C) $t \leq \rho(\mu_X + \mu_Y)$
D) $t \leq \rho(\mu_X - \mu_Y)$ E) All real numbers t

The solution can be found below.

Question 9 Solution

Standardizing X and Y, we have $\frac{X-\mu_X}{\sigma_X}$ has a standard normal distribution, as does $\frac{Y-\mu_Y}{\sigma_Y}$.
Then $F_X(t) = P(X \leq t) = P(\frac{X-\mu_X}{\sigma_X} \leq \frac{t-\mu_X}{\sigma_X}) = \Phi(\frac{t-\mu_X}{\sigma_X})$, and similarly, $F_Y(t) = \Phi(\frac{t-\mu_Y}{\sigma_Y})$.
Then $F_X(t) \geq F_Y(t)$ if $\Phi(\frac{t-\mu_X}{\sigma_X}) \geq \Phi(\frac{t-\mu_Y}{\sigma_Y})$, which occurs if $\frac{t-\mu_X}{\sigma_X} \geq \frac{t-\mu_Y}{\sigma_Y}$.
This inequality can be written as $t-\mu_X \geq (t-\mu_Y) \cdot \frac{\sigma_X}{\sigma_Y} = 2(t-\mu_Y)$
(since we were given that $\sigma_X = 2\sigma_Y$).
The inequality can be rewritten as $t \leq 2\mu_Y - \mu_X$. Answer: A