EXAM M QUESTIONS OF THE WEEK

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Question 9 - Week of September 19

S is the mixture of two compound Poisson distributions, S_1 and S_2, with mixing weights of $\frac{1}{2}$ each. S_1 has Poisson frequency N_1 with $\lambda_1 = 1$ and severity $Y_1 = \begin{cases} 1 & \text{prob. } \frac{1}{2} \\ 2 & \text{prob. } \frac{1}{2} \end{cases}$.

S_2 has Poisson frequency N_2 with $\lambda_2 = 1$ and severity $Y_2 = \begin{cases} 1 & \text{prob. } \frac{1}{3} \\ 2 & \text{prob. } \frac{2}{3} \end{cases}$.

S_3 has a compound Poisson distribution with frequency N_3 with $\lambda = 1$ and severity Y which is a mixture of

$X_1 = \begin{cases} 1 & \text{prob. } \frac{1}{2} \\ 2 & \text{prob. } \frac{1}{2} \end{cases}$ and $X_2 = \begin{cases} 1 & \text{prob. } \frac{1}{3} \\ 2 & \text{prob. } \frac{2}{3} \end{cases}$. with mixing weights of $\frac{1}{2}$ each.

All N's and Y's and X's are independent of one another.

(a) Find $E[S]$, $Var[S]$, $E[S_3]$ and $Var[S_3]$.

(b) Find $P[S = 0]$, $P[S = 1]$, $P[S_3 = 0]$ and $P[S_3 = 1]$.

The solution can be found below.

Question 9 Solution

(a) $E[S] = \frac{1}{2}(E[S_1] + E[S_2]) = \frac{1}{2}[(1)(\frac{3}{2}) + (1)(\frac{5}{3})] = \frac{19}{12}$
$E[S_3] = (1)(\frac{1}{2})[(1)(\frac{3}{2}) + (1)(\frac{5}{3})] = \frac{19}{12}$

$Var[S] = E[Var[S|type]] + Var[E[S|type]]$
$Var[S|type\,1] = (1)(\frac{5}{2})$, $Var[S|type\,2] = (1)(3) \rightarrow E[Var[S|type]] = \frac{1}{2}[\frac{5}{2} + 3] = \frac{11}{4}$.
$E[S|type\,1] = (1)(\frac{3}{2})$, $E[S|type\,2] = (1)(\frac{5}{3}) \rightarrow Var[E[S|type]] = (\frac{3}{2} - \frac{5}{3})^2(\frac{1}{2})(\frac{1}{2}) = \frac{1}{144}$
$Var[S] = \frac{11}{4} + \frac{1}{144} = \frac{397}{144}$.

Alternatively,
$E[S^2] - (E[S])^2 = E[S^2] - (\frac{19}{12})^2$, $E[S^2] = \frac{1}{2}(E[S_1^2] + E[S_2^2])$
$= \frac{1}{2}[Var[S_1] + E[S_1]^2 + Var[S_2] + E[S_2]^2]$
$= \frac{1}{2}[\frac{5}{2} + \frac{9}{4} + 3 + \frac{25}{9}] = \frac{379}{72}$.
$Var[S] = \frac{379}{72} - (\frac{19}{12})^2 = \frac{397}{144}$.

$Var[S_3] = (1)(\frac{1}{2})[\frac{5}{2} + 3] = \frac{11}{4}$.

(b) $P[S=0] = \frac{1}{2}(P[S_1=0] + P[S_2=0]) = \frac{1}{2}(P[N_1=0] + P[N_2=0]) = \frac{1}{2}(e^{-1} + e^{-1})$.
$P[S_3=0] = P[N=0] = e^{-1}$.

$P[S=1] = \frac{1}{2}(P[S_1=1] + P[S_2=1])$
$= \frac{1}{2}(P[N_1=1]\cdot P[Y_1=1] + P[S_2=1]\cdot P[Y_2=1]) = \frac{1}{2}(e^{-1}\cdot\frac{1}{2} + e^{-1}\cdot\frac{1}{3}) = \frac{5}{12}e^{-1}$

$P[S_3=1] = P[N_3=1]\cdot P[X=1] = e^{-1}(\frac{1}{2})[\frac{1}{2} + \frac{1}{3}] = \frac{5}{12}e^{-1}$.