# EXAM M QUESTIONS OF THE WEEK

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## Week of September 18/06

A 3-decrement employee benefit model for death, disability and withdrawal has the following assumptions:

- death and disability have constant forces of decrement, and
- withdrawals occur at the end of each year of age.

A one year term insurance to $(x)$ pays $100,000 at moment of death if death occurs during the year, and $25,000 at the moment of disability if disability occurs during the year. There is also a payment of $10,000 at the end of the year if $(x)$ withdraws at that time.

You are given: $\delta = .10$ , $\mu_x^{(death)}(t) = .05$ , $\mu_x^{(disability)}(t) = .10$ , $q_x'^{(withdrawal)} = .25$ .

Find the actuarial present value of all benefits for the one year policy.

**The solution can be found below.**

## Week of September 18/06 - Solution

Since the withdrawal decrement occurs at the end of the year, we have

$${}_tp_x^{(\tau)} = {}_tp_x'^{(de)} \cdot {}_tp_x'^{(dis)} \cdot {}_tp_x'^{(with)} = e^{-.05t} \cdot e^{-.1t} \cdot 1 = e^{-.15t} \text{ for } 0 < t < 1 .$$

The APV of the death benefit is
$100,000\int_0^1 e^{-\delta t} \cdot {}_tp_x^{(\tau)} \cdot \mu_x^{(death)}(t)\,dt = 100,000\int_0^1 e^{-.1t} \cdot e^{-.15t} \cdot (.05)\,dt = 4424$ .

The APV of the disability benefit is
$25,000\int_0^1 e^{-\delta t} \cdot {}_tp_x^{(\tau)} \cdot \mu_x^{(dis)}(t)\,dt = 25,000\int_0^1 e^{-.1t} \cdot e^{-.15t} \cdot (.1)\,dt = 2212$
(note that this is $\frac{1}{4} \cdot 2 = \frac{1}{2}$ as large as the integral for the death decrement).

Since withdrawal occurs at the end of the year, the withdrawal probability is
$q_x^{(w)} = p_x'^{(death)} \cdot p_x'^{(dis)} \cdot q_x'^{(with)} = e^{-.05} \cdot e^{-.1} \cdot (.25) = .215177$ .
The APV of the withdrawal benefit is
$10,000e^{-\delta} \cdot q_x^{(with)} = 10,000e^{-.1} \cdot (.215177) = 1947$.