

EXAM FM QUESTIONS OF THE WEEK

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Question 8 - Week of September 12

A callable bond with face amount 1000 and annual coupons of 40 matures in 28 years, but it is callable at the end of 25, 26 or 27 years. The redemption value of the bond is based on the following schedule:

Year Called	Redemption Value
25	1000
26	$1000(1 + r)$
27	$1000(1 + r)^2$
28	$1000(1 + r)^3$

The purchaser wishes to realize an annual effective yield of at least 4.5%.

Find the minimum value of r for which the sale price is at least 925.86.

The solution can be found below.

Question 8 Solution

If redemption is at the end of 25 years, the bond price is

$$1000v^{25} + 40a_{\overline{25}|.045} = 925.86 .$$

If the price based on later redemption is lower, the bond purchaser will only purchase at the lower price. Therefore, r must be large enough so that the price based on other redemption dates is at least 925.86.

Price based on redemption at 26 years is

$$1000(1+r)v^{26} + 40a_{\overline{26}|.045} = 318.40(1+r) + 605.86 .$$

Price based on redemption at 27 years is

$$1000(1+r)^2v^{27} + 40a_{\overline{27}|.045} = 304.69(1+r)^2 + 618.05 .$$

Price based on redemption at 28 years is

$$1000(1+r)^3v^{28} + 40a_{\overline{28}|.045} = 291.57(1+r)^3 + 629.71 .$$

In order for 925.86 to be the smallest of these prices, we must have

(i) $318.40(1+r) + 605.86 > 925.86 \rightarrow r > .005025$ and

(ii) $304.69(1+r)^2 + 618.05 > 925.86 \rightarrow r > .005107$ and

(iii) $291.57(1+r)^3 + 629.71 > 925.86 \rightarrow r > .005209$.

Therefore, we must have $r > .005209$ in order for the purchase price to be at least 925.86.