EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of September 11/06

The progression of a disease is being studied. Patients with the disease will either be cured (decrement 1), or they die before being cured (decrement 2). A study of the disease is made using two multiple decrement model. It is assumed that all individuals in the study begin with a mild form and each year, an individual either continues next year with the mild form, or the individual is cured (decrement 1), or the individual dies during the year before being cured (decrement 2). For the purpose of the study, if an individual is cured, that individual will not return to the decrement group. The following table summarizes the behavior of a group at age x who all have the mild form.

Age	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
x	1000	200	50
$x+1$	750		

In the analysis, it is assumed that the decrements are uniformly distributed in the multiple decrement table. A new medical procedure is discovered that will increase the force of decrement due to being cured (decrement 1) by 50% and will reduce the force of decrement due to dying before being cured by 25%. With $\ell_x^{(\tau)} = 1000$, find $d_x^{(1)}$ and $d_x^{(2)}$ based on the revised forces of decrement.

The solution can be found below.

Week of September 11/06 - Solution

If $\mu_x^{(1)}(t)$ is multiplied by 1.5, then the new $p_x^{\prime(1)}$ is $[\text{old } p_x^{\prime(1)}]^{1.5}$,
since $p_x^{\prime(1)} = exp[-\int_0^1 \mu_x^{(1)}(t)\, dt]$.

Similarly, if $\mu_x^{(2)}(t)$ is reduced by 25%, then new $p_x^{\prime(2)}$ is $[\text{old } p_x^{\prime(2)}]^{.75}$.

Under UDD, $p_x^{\prime(1)} = [p_x^{(\tau)}]^{q_x^{(1)}/q_x^{(\tau)}}$ and $p_x^{\prime(2)} = [p_x^{(\tau)}]^{q_x^{(2)}/q_x^{(\tau)}}$.

With the original decrement table, we have $q_x^{(1)} = .2$, $q_x^{(2)} = .05$
and $q_x^{(\tau)} = .25$. Then $p_x^{\prime(1)} = (.75)^{.2/.25} = .794418$
and $p_x^{\prime(2)} = (.75)^{.05/.25} = .944088$.

Under the decrement table based on the new medical procedure, we have
$p_x^{\prime(1)} = (.794418)^{1.5} = .708066$, $p_x^{\prime(2)} = (.944088)^{.75} = .957766$,
and $p_x^{(\tau)} = p_x^{\prime(1)} \cdot p_x^{\prime(2)} = .67816$.
Based on these revised absolute rates, the multiple decrement probabilities will be
$q_x^{(1)} = \frac{ln\, p_x^{\prime(1)}}{ln\, p_x^{(\tau)}} \cdot q_x^{(\tau)} = .2861$, $q_x^{(2)} = \frac{ln\, p_x^{\prime(2)}}{ln\, p_x^{(\tau)}} = .0358$, and $q_x^{(\tau)} = .3219$.

The revised table is

Age	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
x	1000	286	36
$x+1$	678		