

# EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2007

## Week of September 3/07

A particular large calculus class has two term tests and a final exam.

Students are not allowed to drop the course before the first term test.

Class records for past years show the following:

- 80% of students pass the first test
- 30% of students who fail the first term test drop the course before the second test
- 10% of students who pass the first term test drop the course before the second test
- 90% of students who pass the first term test and take the second test pass the second test
- 80% of students who fail the first term test and take the second test pass the second test
- 50% of students who fail the second term test drop the course before the final exam
- none of students who pass the second term test drop the course before the final exam.

Find the fraction of students who drop the course.

**The solution can be found below.**

## Week of September 3/07 - Solution

A student can drop the course after the first test but before the second test.

The fraction of the original group of students that drop the course after the first test but before the second test is

$$\begin{aligned} &P[\text{drop after 1st test but before 2nd test}] \\ &= P[\text{drop after 1st test but before 2nd test} \cap \text{pass 1st test}] \\ &\quad + P[\text{drop after 1st test but before 2nd test} \cap \text{fail 1st test}] \\ &= P[\text{drop after 1st test but before 2nd test} \mid \text{pass 1st test}] \cdot P[\text{pass 1st test}] \\ &\quad + P[\text{drop after 1st test but before 2nd test} \mid \text{fail 1st test}] \cdot P[\text{fail 1st test}] \\ &= (.1)(.8) + (.3)(.2) = .14 \end{aligned}$$

A student can drop the course after the second test but before the final exam.

$$\begin{aligned} &P[\text{drop after 2nd test but before final exam}] \\ &= P[\text{drop after 2nd test but before final exam} \cap \text{pass 1st test} \cap \text{take 2nd test} \cap \text{pass 2nd test}] \\ &\quad + P[\text{drop after 2nd test but before final exam} \cap \text{pass 1st test} \cap \text{take 2nd test} \cap \text{fail 2nd test}] \\ &P[\text{drop after 2nd test but before final exam} \cap \text{fail 1st test} \cap \text{take 2nd test} \cap \text{pass 2nd test}] \\ &\quad + P[\text{drop after 2nd test but before final exam} \cap \text{fail 1st test} \cap \text{take 2nd test} \cap \text{fail 2nd test}] \end{aligned}$$

We find these probabilities in the following way:

$$\begin{aligned} &P[\text{drop after 2nd test but before final exam} \cap \text{fail 1st test} \cap \text{take 2nd test} \cap \text{fail 2nd test}] \\ &= P[\text{drop after 2nd test but before final exam} \mid \text{fail 1st test} \cap \text{take 2nd test} \cap \text{fail 2nd test}] \\ &\quad \times P[\text{fail 2nd test} \mid \text{take 2nd test} \cap \text{fail 1st test}] \times P[\text{take 2nd test} \mid \text{fail 1st test}] \times P[\text{fail 1st test}] \\ &= (.5)(.2)(.7)(.2) = .014 . \end{aligned}$$

Similarly,

$$\begin{aligned} &P[\text{drop after 2nd test but before final exam} \cap \text{pass 1st test} \cap \text{take 2nd test} \cap \text{fail 2nd test}] \\ &= (.5)(.1)(.9)(.8) = .036 . \end{aligned}$$

$P[\text{drop after 2nd test but before final exam} \cap \text{fail 1st test} \cap \text{take 2nd test} \cap \text{pass 2nd test}]$  and  $P[\text{drop after 2nd test but before final exam} \cap \text{pass 1st test} \cap \text{take 2nd test} \cap \text{pass 2nd test}]$  are both 0, since anyone who passes the 2nd test does not drop the course.

The probability of dropping the course is  $.14 + .014 + .036 = .19$ .