EXAM P QUESTIONS OF THE WEEK

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Week of September 24/07

X has a Poisson distribution with a mean of 1, so the probability function for X is

$$P(X = x) = \frac{e^{-1}}{x!} \text{ for } x = 0, 1, 2, ...$$

Y is a new random variable on the non-negative integers. The probability function of Y is related to that of X as follows. A number α is given, with $0 < \alpha < 1$.

$$P(Y = 0) = \alpha \ , \ P(Y = x) = c \cdot P(X = x) \text{ for } x = 1, 2, ...$$

The number c is found so that Y satisfies the requirement for being a random variable $\sum_{x=0}^{\infty} P(Y = x) = 1$.

Find the mean of Y in terms of α and e.

The solution can be found below.

Week of September 24/07 - Solution

Since $\sum_{x=0}^{\infty} P(X=x)=1$, it follows that $\sum_{x=1}^{\infty} P(X=x)=1-P(X=0)=1-e^{-1}$.

Then, $\sum_{x=1}^{\infty} P(Y=x)=c \cdot \sum_{x=1}^{\infty} P(X=x)=c(1-e^{-1})$.
But it is also true that $\sum_{x=1}^{\infty} P(Y=x)=1-P(Y=0)=1-\alpha$.

Therefore, $c(1-e^{-1})=1-\alpha$, so that $c=\frac{1-\alpha}{1-e^{-1}}$.

The mean of Y is
$E[Y]=\sum_{x=0}^{\infty} x \cdot P(Y=x)=\sum_{x=1}^{\infty} x \cdot P(Y=x)=\sum_{x=1}^{\infty} x \cdot c \cdot P(X=x)=c \cdot \sum_{x=o}^{\infty} x \cdot P(X=x)$
$=c \cdot E[X]=c=\frac{1-\alpha}{1-e^{-1}}$.