EXAM P QUESTIONS OF THE WEEK

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Week of October 1/07

The Toronto Maple Leafs have two suppliers for hockey sticks, Crosscheck Lumber, and Sticks R Us. The Leafs get equal numbers of sticks from each supplier, and since the team logo is branded on every stick, after the sticks are delivered, it is not possible to tell what supplier provided any particular stick. The team estimates that on average, 10% of the sticks from Crosscheck lumber are defective and 20% of the sticks from Sticks R Us are defective. A Leaf player examines 10 sticks from a recent shipment from a supplier but doesn't know who the supplier was. The player finds 2 defective sticks out of the 10 sticks. Find the probability that the supplier of those sticks was Crosscheck Lumber.

The solution can be found below.

Week of October 1/07 - Solution

We define the following events:
C - shipment is from Crosscheck Lumber
S - shipment is from Sticks R Us
$2D$ - 2 sticks are defective

We wish to find $P(C|2D)$. This is $\frac{P(C \cap 2D)}{P(2D)}$.

The numerator can be formulated as $P(2D|C) \cdot P(C)$.
We are given that $P(C) = .5$. For a shipment from Crosscheck Lumber, the number of sticks that are defective in a batch of 10 sticks has a binomial distribution with $n = 10$ and $p = .1$ (prob. of a particular stick being defective).
Therefore, $P(2D|C) = \binom{10}{2}(.1)^2(.9)^8 = .193710$.
The numerator is $P(C \cap 2D) = (.193710)(.5) = .096855$.

The denominator can be formulated as $P(2D) = P(C \cap 2D) + P(S \cap 2D)$ since the shipment must be either C or S. We find $P(S \cap 2D)$ in the same way as $P(C \cap 2D)$.
$P(S \cap 2D) = P(2D|S) \cdot P(S) = \binom{10}{2}(.2)^2(.8)^8 \cdot (.5) = .150995$.

Then, $P(C|2D) = \frac{P(C \cap 2D)}{P(2D)} = \frac{P(C \cap 2D)}{P(C \cap 2D) + P(S \cap 2D)} = \frac{.096855}{.096855 + .150995} = .39$.