EXAM P QUESTION OF THE WEEK

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The loss random variable X has an exponential distribution with a mean of 100.

When a loss occurs, an insurance policy pays the part of the loss that is above 20.
If the loss is below 20, the insurance policy does not pay anything.

Y denotes the amount paid by the insurer when a loss occurs. Find $Var(Y)$.

The solution can be found below.

Week of May 5/08 - Solution

The pdf of X is $f(x) = .01e^{-.01x}$.

$$Y = \begin{cases} 0 & \text{if } X \leq 20 \\ X - 20 & \text{if } X > 20 \end{cases} \ .$$

$Var(Y) = E(Y^2) - [E(Y)]^2$.

$E(Y) = \int_{20}^{\infty} (x - 20) \cdot (.01)e^{-.01x}\, dx$
Substituting $u = x - 20$ results in
$E(Y) = \int_0^{\infty} u \cdot .01e^{-.01(u+20)}\, du = e^{-.2} \cdot \int_0^{\infty} u \cdot .01e^{-.01u}\, du = e^{-.2} \cdot 100 = 100e^{-.2}$,
since $\int_0^{\infty} u \cdot .01e^{-.01u}\, du$ is the mean of an exponential random variable with mean 100.

$E(Y^2) = \int_{20}^{\infty} (x - 20)^2 \cdot (.01)e^{-.01x}\, dx$
Substituting $u = x - 20$ results in
$E(Y^2) = \int_0^{\infty} u^2 \cdot .01e^{-.01(u+20)}\, du = e^{-.2} \cdot \int_0^{\infty} u^2 \cdot .01e^{-.01u}\, du = e^{-.2} \cdot 2 \cdot 100^2 = 20,000e^{-.2}$,
since $\int_0^{\infty} u^2 \cdot .01e^{-.01u}\, du$ is the second moment of an exponential random variable with mean 100.

$Var(Y) = 20,000e^{-.2} - (100e^{-.2})^2 = 9671$.