EXAM P QUESTIONS OF THE WEEK

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Week of May 14/07

A statistician for the National Hockey League has created a model for the number of goals scored per 60-minute game by the Ottawa Senators and the Buffalo Sabres. According to the model, the number of goals scored per game by the Senators has a geometric distribution, $X_{OTT} = 0, 1, 2, ...$ with a mean of 3.5. The model also has a similar geometric distribution for the number of goals scored per 60-minute game by the Sabres, X_{BUF}, with a mean of 3.0. Assuming that X_{OTT} and X_{BUF} are independent, find the probability that Buffalo wins the game in 60 minutes by at least 2 goals.

The solution can be found below.

Week of May 14/07 - Solution

The geometric distribution $X = 0, 1, 2, ...$ has probability function $P[X = k] = (1 - p)^k p$ and has mean $\frac{1-p}{p}$. For the Senators, we have $\frac{1-p_{OTT}}{p_{OTT}} = 3.5$, so that $p_{OTT} = \frac{1}{4.5}$.
For the Sabres, we have $\frac{1-p_{BUF}}{p_{BUF}} = 3$, so that $p_{OTT} = \frac{1}{4}$.

$P[X_{OTT}] = k = (1 - \frac{1}{4.5})^k (\frac{1}{4.5}) = (\frac{7}{9})^k (\frac{2}{9})$ and $P[X_{BUF} = k] = (1 - \frac{1}{4})^k (\frac{1}{4}) = (\frac{3}{4})^k (\frac{1}{4})$.

$P[X_{BUF} \geq n] = \sum_{k=n}^{\infty} (\frac{3}{4})^k (\frac{1}{4}) = (\frac{3}{4})^n$.

$P[X_{BUF} \geq X_{OTT} + 2]$
$= P[X_{BUF} \geq 2 | X_{OTT} = 0] \cdot P[X_{OTT} = 0] + P[X_{BUF} \geq 3 | X_{OTT} = 1] \cdot P[X_{OTT} = 1] + \cdots$
$= \sum_{n=0}^{\infty} P[X_{BUF} \geq n + 2 | X_{OTT} = n] \cdot P[X_{OTT} = n]$
$= \sum_{n=0}^{\infty} P[X_{BUF} \geq n + 2] \cdot P[X_{OTT} = n]$ (becuase of independence of X_{BUF} and X_{OTT})
$= \sum_{n=0}^{\infty} (\frac{3}{4})^{n+2} (\frac{7}{9})^n (\frac{2}{9}) = (\frac{3}{4})^2 (\frac{2}{9}) \sum_{n=0}^{\infty} (\frac{3}{4} \cdot \frac{7}{9})^n = \frac{1}{8} \cdot \frac{1}{1 - \frac{7}{12}} = .30$.