

EXAM P QUESTION OF THE WEEK

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Y is a 2-point random variable with $P(Y = 0) = \frac{1}{3}$ and $P(Y = 1) = \frac{2}{3}$.

The random variable X satisfies the following properties:

$$E(X|Y = 0) = 2, \quad E(X|Y = 1) = 4, \quad \text{Var}(X|Y = 0) = 1, \quad \text{Var}(X|Y = 1) = 4$$

Find the unconditional variance of X .

The solution can be found below.

Week of May 12/08 - Solution

We use the following rule of probability that is valid for any two random variables U and W

$$\text{Var}(U) = E[\text{Var}(U|W)] + \text{Var}[E(W|U)]$$

Then $\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}[E(X|Y)]$

$$\begin{aligned} E[\text{Var}(X|Y)] &= \text{Var}(X|Y=0) \cdot P(Y=0) + \text{Var}(X|Y=1) \cdot P(Y=1) \\ &= (1)\left(\frac{1}{3}\right) + (4)\left(\frac{2}{3}\right) = 3 \quad \text{and} \end{aligned}$$

$$\text{Var}[E(X|Y)] = \text{Var} \begin{cases} 2 = E(X|Y=0) & \text{prob. } \frac{1}{3} \\ 4 = E(X|Y=1) & \text{prob. } \frac{2}{3} \end{cases} = (2-4)^2\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{9}.$$

$$\text{Var}(X) = 3 + \frac{8}{9} = \frac{35}{9}.$$