

EXAM P QUESTIONS OF THE WEEK

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Week of March 5/07

The Toronto Blue Jays baseball team holds a Children's Hospital Day. The Blue Jays will donate \$100,000 for each home run hit after the 2nd home run in the game. The team's model for the number of home runs hit in the game is Poisson with a mean of 4. Find the expected amount that the Blue Jays will donate.

The solution can be found below.

Week of March 5/07 - Solution

N denotes the number of home runs hit in the game. $E[N] = 4$ and N has a Poisson distribution. The amount donated X (multiples of 100,000) can be summarized as follows:

Define Y to be $Y = N - X$.

| | | | | | | | |
|-----|---|---|---|---|---|---|-----|
| N | 0 | 1 | 2 | 3 | 4 | 5 | ... |
| X | 0 | 0 | 0 | 1 | 2 | 3 | ... |
| Y | 0 | 1 | 2 | 2 | 2 | 2 | ... |

We know that $X + Y = N$ so that $E[X] + E[Y] = E[N] = 4$.

But we also can see that Y can only be 0, 1 or 2, and

$$P(Y = 0) = P(N = 0) = e^{-4}, \quad P(Y = 1) = P(N = 1) = 4e^{-4}$$

$$\text{and } P(Y = 2) = P(N \geq 2) = 1 - P(N = 0, 1) = 1 - 5e^{-4}.$$

Therefore,

$$E[X] = 4 - E[Y] = 4 - (1)(4e^{-4}) - (2)[1 - 5e^{-4}] = 2.11$$

and the expected amount paid by the Blue Jays is 211,000.