EXAM P QUESTION OF THE WEEK

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Week of March 24/08

X is a Poisson random variable with mean λ and Y is a Poisson random variable with mean $\lambda + 1$.

X and Y are independent random variables.

You are given that $E(X^3) = 22$ and $E(Y^3) = 57$.

Determine $P(X = Y = 0)$.

The solution can be found below.

Week of March 24/08 - Solution

The moment generating function of X is $M_X(t) = E(e^{tX}) = e^{\lambda(e^t - 1)}$,
and the 3rd moment of X is
$\frac{d}{dt^3} M_X(t)\Big|_{t=0} = e^{\lambda(e^t-1)} \cdot [\lambda^3 e^{3t} + 3\lambda^2 e^{2t} + \lambda e^t]\Big|_{t=0}$
$= \lambda^3 + 3\lambda^2 + \lambda = 22$.

The 3rd moment of Y is
$\frac{d}{dt^3} M_Y(t)\Big|_{t=0} = e^{2(\lambda+1)(e^t-1)} \cdot [(\lambda+1)^3 e^{3t} + 3(\lambda+1)^2 e^{2t} + (\lambda+1)\lambda e^t]$
$= (\lambda+1)^3 + 3(\lambda+1)^2 + \lambda + 1 = 57$.

Then
$(\lambda+1)^3 + 3(\lambda+1)^2 + \lambda + 1 - (\lambda^3 + 3\lambda^2 + \lambda) = 57 - 22 = 35$,
so that $3\lambda^2 + 9\lambda + 5 = 35$, or equivalently, $\lambda^2 + 3\lambda - 10 = 0$.
Solving for λ results in $\lambda = 2$ or -5 . We reject the negative root since the Poisson mean must be positive.

Since X and Y are independent, we have
$P(X = Y = 0) = P(X = 0) \cdot P(Y = 0) = e^{-\lambda} \cdot e^{-(\lambda+1)} = e^{-5}$.