

EXAM P QUESTIONS OF THE WEEK

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Week of March 12/07

Let X be a random variable with distribution function

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x}{8} & \text{for } 0 \leq x < 1 \\ \frac{1}{4} + \frac{x}{8} & \text{for } 1 \leq x < 2 \\ \frac{3}{4} + \frac{x}{12} & \text{for } 2 \leq x < 3 \\ 1 & \text{for } x \geq 3 \end{cases} . \quad \text{Calculate } P[1 \leq X \leq 2] .$$

- A) $\frac{1}{8}$ B) $\frac{3}{8}$ C) $\frac{7}{16}$ D) $\frac{13}{24}$ E) $\frac{19}{24}$

The solution can be found below.

Week of March 12/07 - Solution

From the definition of $F(x)$, we see that as $x \rightarrow 1^-$ (limit from the left) $F(x) \rightarrow \frac{1}{8}$, but at $x = 1$, we have $F(1) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$. This shows that X has a continuous distribution for $x < 1$, but there is a discrete point of probability at $x = 1$, and the probability at that point is $P[X = 1] = \frac{1}{4}$, which is the size of the "jump" of $F(x)$ from $\frac{1}{8}$ to $\frac{3}{8}$ as x reaches $x = 1$.

$$\begin{aligned} P[1 \leq X \leq 2] &= P[1 < X \leq 2] + P[X = 1] = F(2) - F(1) + P[X = 1] \\ &= \frac{11}{12} - \frac{3}{8} + \frac{1}{4} = \frac{19}{24}. \end{aligned}$$

Answer: E