# EXAM P QUESTIONS OF THE WEEK

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## **Week of July 30/07**

A fair coin is tossed 100 times. The tosses are independent of one another.
The number of heads tossed is $X$.

It is desired to find the smallest integer value $k$ which satisfies the probability relationship
$P(50 - k \leq X \leq 50 + k) \geq .95$ .

Find $k$ by applying the normal approximation with integer correction to the distribution of $X$.

**The solution can be found below.**

## Week of July 30/07 - Solution

$X$ has a binomial distribution with 100 trials and probability $\frac{1}{2}$ of success. The expected number of heads is $100(\frac{1}{2}) = 50$ and the variance of the number of heads is $100(\frac{1}{2})(\frac{1}{2}) = 25$ .
Using the normal approximation with integer correction, we want to satisfy the relationship
$P(50 - k - .5 \leq X \leq 50 + k + .5) \geq .95$ .

Applying the normal approximation, we have
$P(50 - k - .5 \leq X \leq 50 + k + .5) = P(\frac{-k-.5}{\sqrt{25}} \leq \frac{X-50}{\sqrt{25}} \leq \frac{k+.5}{\sqrt{25}}) = P(-c \leq Z \leq c)$ ,
where $Z$ is standard normal. In order for this probability to be at least .95, it must be true that $\Phi(c) \geq .975$ . This is true because we want to eliminate less than .025 probability from the left and right side of $Z$.

From the standard normal table, $\Phi(1.96) = .975$ , and therefore, we must have $c \geq 1.96$ .
Then, $\frac{k+.5}{5} \geq 1.96 \rightarrow k \geq 9.3$ . The smallest integer is $k = 10$ .

Using the normal approximation, $P(40 \leq X \leq 60) \geq .95$ but $P(41 \leq X \leq 59) < .95$ .