EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2007

Week of July 23/07

n fair six-sided dice are tossed independently of one another.

Find the probability that the sum is even.

A) $\frac{1}{2}-\frac{(n-1)(n-2)(n-3)}{6n^3}$ B) $\frac{1}{2}-\frac{(n-1)(n-2)}{6n^2}$ C) $\frac{1}{2}$

D) $\frac{1}{2}+\frac{(n-1)(n-2)(n-3)}{6n^3}$ E) $\frac{1}{2}+\frac{(n-1)(n-2)}{6n^2}$

The solution can be found below.

Week of July 23/07 - Solution

The probability of an even outcome when tossing a single $(n = 1)$ die is $\frac{1}{2}$.

The probabilities for the sum when tossing two dice are

Sum	2	3	4	5	6	7	8	9	10	11	12
Prob	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The probability that the sum is even is $\frac{1}{36} + \frac{3}{36} + \frac{5}{36} + \frac{5}{36} + \frac{3}{36} + \frac{1}{36} = \frac{18}{36} = \frac{1}{2}$.

To see that the probability is always $\frac{1}{2}$, suppose that E_{n-1} is the event that sum of the first $n-1$ tosses is even. Then in order for the sum of the n dice to be even, we must have either E_{n-1} occurring and the n-th toss is even, or E'_{n-1} occurring (complement) and the n-th toss is odd. Because of independence of the tosses, we get

$P(\text{sum of } n \text{ tosses is even}) = P(E_{n-1})(\frac{1}{2}) + P(E'_{n-1})(\frac{1}{2}) = (\frac{1}{2})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{2}) = \frac{1}{2}$.

Since $P(E_1) = \frac{1}{2}$, it follows that $P(E_k) = \frac{1}{2}$ for any k.