EXAM P QUESTIONS OF THE WEEK

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Week of January 7/08

You are given the following:

- X_1 has a binomial distribution with a mean of 2 and a variance of 1.
- X_2 has a Poisson distribution with a variance of 2.
- X_1 and X_2 are independent.
- $Y = X_1 + X_2$.

What is $P(Y < 3)$?

A) $\frac{11}{16}e^{-2}$ B) $\frac{15}{16}e^{-2}$ C) $\frac{19}{16}e^{-2}$ D) $\frac{23}{16}e^{-2}$ E) $\frac{27}{16}e^{-2}$

The solution can be found below.

Week of January 7/08 - Solution

X_1 is binomial with $np = 2$ and $np(1-p) = 1$.
It follows that $1 - p = \frac{1}{2}$, and $p = \frac{1}{2}$, and $n = 4$.
The probability function of X_1 is $P(X_1 = k) = \binom{4}{k}(\frac{1}{2})^k(\frac{1}{2})^{4-k} = \binom{4}{k}(\frac{1}{2})^4$.

The probability function of X_2 is $P(X_2 = j) = \frac{2^j e^{-2}}{j!}$.

$P(Y < 3) = P(Y = 0) + P(Y = 1) + P(Y = 2)$.

$P(Y = 0) = P(X_1 = 0 \cap X_2 = 0) = P(X_1 = 0) \times P(X_2 = 0)$
$= \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!} = \frac{1}{16}e^{-2}$.

$P(Y = 1) = P(X_1 = 0 \cap X_2 = 1) + P(X_1 = 1 \cap X_2 = 0)$
$= P(X_1 = 0) \times P(X_2 = 1) + P(X_1 = 1) \times P(X_2 = 0)$
$= \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^1 e^{-2}}{1!} + \binom{4}{1}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!} = \frac{2}{16}e^{-2} + \frac{4}{16}e^{-2} = \frac{6}{16}e^{-2}$.

$P(Y = 2) = P(X_1 = 0 \cap X_2 = 2) + P(X_1 = 1 \cap X_2 = 1) + P(X_1 = 2 \cap X_2 = 0)$
$= P(X_1 = 0) \times P(X_2 = 2) + P(X_1 = 1) \times P(X_2 = 1) + P(X_1 = 2) \times P(X_2 = 0)$
$= \binom{4}{0}(\frac{1}{2})^4 \times \frac{2^2 e^{-2}}{2!} + \binom{4}{1}(\frac{1}{2})^4 \times \frac{2^1 e^{-2}}{1!} + \binom{4}{2}(\frac{1}{2})^4 \times \frac{2^0 e^{-2}}{0!}$
$= \frac{2}{16}e^{-2} + \frac{8}{16}e^{-2} + \frac{6}{16}e^{-2} = e^{-2}$.

Then, $P(Y < 3) = \frac{1}{16}e^{-2} + \frac{6}{16}e^{-2} + e^{-2} = \frac{23}{16}e^{-2}$. Answer: B