

# EXAM P QUESTIONS OF THE WEEK

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## Week of January 22/07

A loss random variable has an exponential distribution with mean 800.

If an insurer imposes a policy limit of  $u$  on the loss, the insurer will pay a maximum of  $u$  when a loss occurs. The expected payment by the insurer with a policy limit of  $u$  is  $A$ . If instead the insurer imposes a policy limit of  $2u$  on the loss, the expected payment by the insurer will be  $1.2865A$  when a loss occurs. Find  $u$ .

**The solution can be found below.**

## **Week of January 22/07 - Solution**

The exponential distribution with mean  $\theta$  has pdf  $f(t) = \frac{1}{\theta}e^{-t/\theta}$  and cdf  $F(x) = 1 - e^{-x/\theta}$ .

For a non-negative loss random variable  $L$  with cdf  $F(y)$ , if a policy limit of  $u$  is imposed, the expected payment by the insurer when a loss occurs is  $\int_0^u [1 - F(y)] dy$ .

For the exponential loss random variable with mean 800 and with limit  $u$ , the expected amount paid by the insurer when a loss occurs is  $\int_0^u e^{-x/800} dx = 800[1 - e^{-u/800}]$ .

If the limit is  $2u$ , the expected payment by the insurer when a loss occurs is  $800[1 - e^{-2u/800}]$ .

We are given that  $800[1 - e^{-2u/800}] = 1.2865(800[1 - e^{-u/800}])$ .

After canceling 800 and factoring the difference of squares

$$1 - e^{-2u/800} = (1 - e^{-u/800})(1 + e^{-u/800}),$$

this equation becomes  $1 + e^{-u/800} = 1.2865$ , so that  $u = 1000$ .