EXAM P QUESTIONS OF THE WEEK

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Week of January 15/07

A fair 6-sided die with faces numbered 1 to 6 is tossed successively and independently until the total of the faces is at least 14. Find the probability that at least 4 tosses are needed.

The solution can be found below.

Week of January 15/07 - Solution

$P[\text{At least 4 tosses are needed}] = 1 - P[\text{at most 3 tosses are needed}]$.

It is not possible to reach the total of 14 on 1 or 2 tosses.
There are $6 \times 6 \times 6 = 216$ possible sets of 3 consecutive tosses.
The following sets of 3 consecutive tosses result in a total of at least 14 on the faces that turn up.

(a) 3 sixes (6 on each toss) ; 1 set.
(b) 2 sixes and two to five on the other toss ; $4 \times 3 = 12$ sets
(6,6,2 , and 6,2,6 and 2,6,6 , and the same with 3 or 4 or 5 instead of 2).
(c) 1 six and either five-five, or four-five, or four-four, or three-five; $3 + 6 + \; + 3 + 6 = 18$ sets
(6,5,5 or 5,6,5 or 5,5,6, and 6,4,5 in six arrangements, and 6-3-5 in six arrangements).
(d) 0 sixes, and either 3 fives, or 2 fives and 1 four; $1 + 3 = 4$ sets.

Total of $1 + 12 + 18 + 4 = 35$ sets out of 216 possible sets.
Probability is $P[\text{At least 4 tosses are needed}] = 1 - \frac{35}{216} = .838$.