EXAM P QUESTIONS OF THE WEEK

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Week of January 14/08

X has pdf $f(x) = x$ for $0 < x < 1$.
Also, $P(X = 0) = a$ and $P(X = 1) = b$, and $P(X < 0) = P(X > 1) = 0$.

For what value of a is $Var(X)$ maximized?

A) $0 \leq a < .1$ B) $.1 \leq a < .2$ C) $.2 \leq a < .3$ D) $.3 \leq a < .4$ E) $a \geq .4$

The solution can be found below.

Week of January 14/08 - Solution

In order to be a properly defined random variable, we must have

$P(X=0)+P(0<X<1)+P(X=1)=1$, so that

$a+\int_0^1 x\,dx+b=a+\frac{1}{2}+b=1$. Therefore, $a+b=\frac{1}{2}$.

$Var(X)=E(X^2)-[E(X)]^2$.

$E(X)=0\times a\,+\,\int_0^1 x\times x\,dx\,+\,1\times b=\frac{1}{3}+b$, and

$E(X^2)=0\times a^2\,+\,\int_0^1 x^2\times x\,dx\,+\,1^2\times b=\frac{1}{4}+b$.

Then, $Var(X)=\frac{1}{4}+b-(\frac{1}{3}+b)^2=\frac{5}{36}+\frac{b}{3}-b^2$.

$Var(X)$ will be maximized if $\frac{d}{db}[\frac{5}{36}+\frac{b}{3}-b^2]=\frac{1}{3}-2b=0$.
This occurs at $b=\frac{1}{6}$. Then $a=\frac{1}{2}-b=\frac{1}{3}$.

Answer: D