EXAM P QUESTIONS OF THE WEEK

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Week of February 5/07

A model describes the time until a loss occurs, X, and the size of the loss, Y.

X has pdf $f_X(x) = \frac{1}{x^2}$ for $x > 1$.

The conditional distribution of Y given $X = x$ has pdf $f_{Y|X}(y|X = x) = \frac{1}{x}$ for $x < y < 2x$.

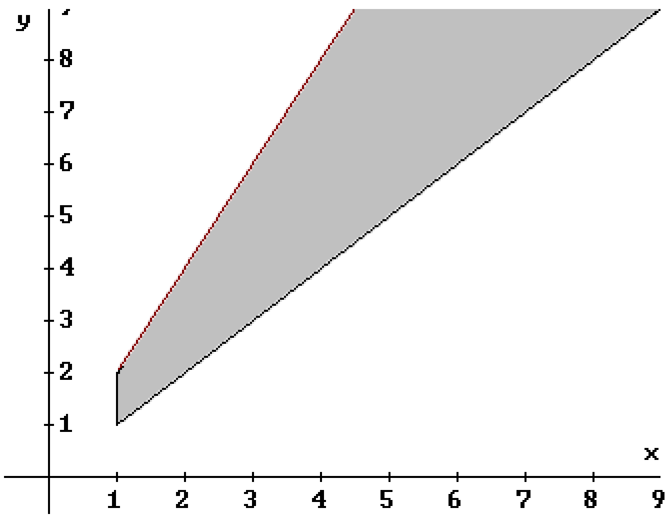
Find pdf of the marginal distribution of Y, $f_Y(y)$.

The solution can be found below.

Week of February 5/07 - Solution

Since $x < y < 2x$, it follows that $\frac{y}{2} < x < y$.
Also, since $x > 1$ it follows that $x > max\{\frac{y}{2}, 1\}$, and $y > 1$.
Therefore, if $1 < y \leq 2$, it follows that $x > 1$, and if $y > 2$ then $x > \frac{y}{2}$.

The joint density of X and Y is $f(x, y) = f(y|x) \cdot f_X(x) = \frac{1}{x} \cdot \frac{1}{x^2} = \frac{1}{x^3}$.
If $1 < y < 2$, then this joint pdf is defined for $1 < x < y$,
and if $y \geq 2$, then this joint pdf is defined for $\frac{y}{2} < x < y$.
The shaded region below is the region of joint density.



The pdf of the marginal distribution of Y is $f_Y(y) = \int f(x, y)\, dx$.
For $1 < y < 2$, we get $f_Y(y) = \int_1^y \frac{1}{x^3}\, dx = \frac{1}{2} - \frac{1}{2y^2}$.
For $y \geq 2$, we get $f_Y(y) = \int_{y/2}^y \frac{1}{x^3}\, dx = \frac{4}{2y^2} - \frac{1}{2y^2} = \frac{3}{2y^2}$.