EXAM P QUESTIONS OF THE WEEK

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Week of August 6/07

The loss random variable X has an exponential distribution with a mean of $\theta > 0$. An insurance policy pays Y, where

$$Y = \begin{cases} \frac{X}{2} & \text{if } X \le \theta \\ X & \text{if } X > \theta \end{cases}$$

Find $E[Y]$.

A) $\frac{\theta}{2}(1 + e^{-1})$ B) $\frac{\theta}{2}(1 + 2e^{-1})$ C) $\frac{\theta}{2}(1 - e^{-1})$

D) $\frac{\theta}{2}(1 - 2e^{-1})$ E) $\theta(1 - e^{-1})$

The solution can be found below.

Week of August 6/07 - Solution

$E[Y] = \int_0^\theta \frac{x}{2} \cdot \frac{1}{\theta} e^{-x/\theta}\, dx \; + \; \int_\theta^\infty x \cdot \frac{1}{\theta} e^{-x/\theta}\, dx$

$= \frac{1}{2}(- xe^{-x/\theta} - \theta e^{-x/\theta}) \Big|_{x=0}^{x=\theta} \; + \; (- xe^{-x/\theta} - \theta e^{-x/\theta}) \Big|_{x=\theta}^{x=\infty}$

$= \frac{\theta}{2}(1 - 2e^{-1}) + 2\theta e^{-1} = \frac{\theta}{2}(1 + 2e^{-1})$

Answer: B