

EXAM P QUESTIONS OF THE WEEK

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Week of August 20/07

A loss random variable X has a continuous uniform distribution on the interval $(0, 100)$.

An insurance policy on the loss pays the full amount of the loss if the loss is less than or equal to 40. If the loss is above 40 but less than or equal to 80, then the insurance pays 40 plus one-half of the loss in excess of 40. If the loss is above 80, the insurance pays 60. If Y denotes the amount paid by the insurance when a loss occurs, find the variance of Y .

The solution can be found below.

Week of August 20/07 - Solution

$$Y = \begin{cases} X & X \leq 40 \\ 40 + \frac{1}{2}(x - 40) & 40 < X \leq 80 \\ 60 & X > 80 \end{cases}$$

$$\text{Var}[Y] = E[Y^2] - (E[Y])^2.$$

$$\begin{aligned} E[Y] &= \int_0^{40} x \cdot (.01) dx + \int_{40}^{80} (20 + .5x)(.01) dx + \int_{80}^{100} 60(.01) dx \\ &= 8 + 20 + 12 = 40. \end{aligned}$$

$$\begin{aligned} E[Y^2] &= \int_0^{40} x^2 \cdot (.01) dx + \int_{40}^{80} (20 + .5x)^2(.01) dx + \int_{80}^{100} 60^2(.01) dx \\ &= \frac{640}{3} + \frac{3040}{3} + 720 = \frac{5840}{3}. \end{aligned}$$

$$\text{Var}[Y] = \frac{5840}{3} - 40^2 = \frac{1040}{3}.$$