EXAM P QUESTIONS OF THE WEEK

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Week of April 23/07

Let X and Y be discrete random variables with joint probability function $f(x, y)$ given by the following table:

		x			
		2	3	4	5
	0	.05	.05	.15	.05
y	1	.40	0	0	0
	2	.05	.15	.10	0

Calculate $Cov[X-Y, X+Y]$.

A) Less than -1 B) At least -1 . but less than 0

C) At least 0 but less than 1 D) At least 1 but less than 2

E) At least 2

The solution can be found below.

Week of April 23/07 - Solution

$Cov[X - Y, X + Y] = Cov[X, X] + Cov[X, Y] - Cov[Y, X] - Cov[Y, Y]$
$= Var[X] - Var[Y]$

The marginal distribution of X has probability function
$P(X = 2) = .5$, $P(X = 3) = .2$, $P(X = 4) = .25$, $P(X = 5) = .05$.
$E[X] = (2)(.5) + (3)(.2) + (4)(.25) + (5)(.05) = 2.85$.
$E[X^2] = (4)(.5) + (9)(.2) + (16)(.25) + (25)(.05) = 9.05$.
$Var[X] = E[X^2] = (E[X])^2 = 9.05 - 2.85^2 = .9275$.

The marginal distribution of Y has probability function
$P(Y = 0) = .3$, $P(Y = 1) = .4$, $P(Y = 2) = .3$.
$E[Y] = (1)(.4) + (2)(.3) = 1.0$.
$E[Y^2] = (1)(.4) + (4)(.3) = 1.6$.
$Var[Y] = E[Y^2] = (E[Y])^2 = 1.6 - 1^2 = .6$.

$Cov[X - Y, X + Y] = Var[X] - Var[Y] = .9275 - .6 = .3275$.

Answer: C