EXAM P QUESTION OF THE WEEK

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A loss distribution has pdf $f(x) = \frac{1}{x^2}$ for $x > 1$.

An insurer finds that the time in hours it takes to process a loss of amount x has a uniform distribution on the interval $(x^{1/2}\, ,\, 2x^{1/2})$.

Find the expected time it takes to process a claim of random amount.

The solution can be found below.

Week of April 14/08 - Solution

We are given that the conditional distribution of process time T for a claim of size x is

$g(t|x) = \frac{1}{x^{1/2}}$ for $x^{1/2} < t < 2x^{1/2}$.

The joint density of T and X is $f(t,x) = g(t|x) \cdot f(x) = \frac{1}{x^{1/2}} \cdot \frac{1}{x^2} = \frac{1}{x^{5/2}}$
for $x > 1$ and $x^{1/2} < t < 2x^{1/2}$.

The inequalities $x^{1/2} < t < 2x^{1/2}$ and $x > 1$
are equivalent to $\frac{t^2}{4} < x < t^2$. if $t > 2$ and $1 < x < t^2$ if $1 < t \leq 2$.

The density of the marginal distribution of T is
$\int_{t^2/4}^{t^2} \frac{1}{x^{5/2}} \, dx = \frac{14}{3t^3}$ for $t > 2$ and .$\int_1^{t^2} \frac{1}{x^{5/2}} \, dx = \frac{2t^3-2}{3t^3}$ if $1 < t \leq 2$.

The mean of T is $\int_1^2 t \cdot \frac{2t^3-2}{3t^3} \, dt + \int_2^\infty t \cdot \frac{14}{3t^3} \, dt = 3$.

An alternative approach is to use the double expectation rule $E[T] = E[\, E[T|X] \,]$.

From the conditional distribution of T given X we have $E[T|X] = \frac{3X^{1/2}}{2}$ (the mean of the uniform distribution on the interval $(x^{1/2} \, , \, 2x^{1/2})$.
Then, $E[\frac{3X^{1/2}}{2}] = \int_1^\infty \frac{3x^{1/2}}{2} \cdot \frac{1}{x^2} \, dx = \int_1^\infty \frac{3}{2x^{3/2}} \, dx = 3$.