EXAM M QUESTIONS OF THE WEEK

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Week of October 9/06

A population of auto insurance policies consists of three types of policies.
Low risk policies make up 60% of the population, medium risk policies make up 30% of the population, and the other 10% are high risk.

The number of claims per year for a low risk policy has a Poisson distribution with a mean of .2. The number of claims per year for a medium risk policy has a Poisson distribution with a mean of 1. The number of claims per year for a high risk policy has a Poisson distribution with a mean of λ.

For a randomly chosen policy from the population, the variance of the number of claims in a year is 1.1701. Find the expected number of claims per year for a high risk policy.

The solution can be found below.

Week of October 9/06 - Solution

The randomly chosen policy is a mixture of the three policy types.

The expected number of claims per year for a randomly chosen policy is
$(.6)(.2) + (.3)(1) + (.1)\lambda = .42 + .1\lambda$.

The second moment of a Poisson random variable is $E[N^2] = Var[N] + (E[N])^2 = \lambda + \lambda^2$.
The second moment of number of claims for a low risk policy is $.2 + (.2)^2 = .24$.
The second moment of number of claims for a medium risk policy is $1 + (1)^2 = 2$.
The second moment of number of claims for a high risk policy is $\lambda + (\lambda)^2$.

The second moment of the number of claims per year for a randomly chosen policy is
$(.6)(.24) + (.3)(2) + (.1)(\lambda + \lambda^2) = .744 + .1(\lambda + \lambda^2)$.

The variance of the number of claims in a year for a randomly chosen policy is
$.744 + .1(\lambda + \lambda^2) - (.42 + .1\lambda)^2 = .5676 + .016\lambda + .09\lambda^2$.
We are given that this is 1.1701 , so that $.5676 + .016\lambda + .09\lambda^2 = 1.1701$.
This is the quadratic equation $.09\lambda^2 + .016\lambda - .6025 = 0$.
The two roots of the equation are 2.5 and $-$ 2.67. We ignore the negative root.
$\lambda =$ 2.5 .