

EXAM FM QUESTIONS OF THE WEEK

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Week of October 9/06

A portfolio contains the following two bonds

	Bond 1	Bond 2
Face amount	\$10,000	\$10,000
Term	20 years	26 years
Coupon amount	\$800	None
Coupon frequency	Annually	N/A
Modified duration	9.82 years	
Interest rate	8% per year	8% per year
	compounded annually	compounded annually

Find the modified duration of the two bond portfolio.

The solution can be found below.

Week of October 9/06 - Solution

The modified duration of a cashflow is $D_M = -\frac{dP}{di} / P$, where P is the present value of the cashflow. Therefore, the modified duration of bond 1 is $9.82 = -\frac{dP_1}{di} / P_1$.

Bond 2 is a zero-coupon bond. The modified duration of a zero-coupon bond is nv , where n is the time to maturity. Therefore the modified duration of bond 2 is

$$\frac{26}{1.08} = 24.07 = -\frac{dP_2}{di} / P_2.$$

The price of Bond 1 is $10,000v_{.08}^{20} + 800a_{\overline{10}|.08} = 10,000$, and the price of Bond 2 is $10,000v^{26} = 1352$. Therefore, $-\frac{dP_1}{di} = 98,200$ and $-\frac{dP_2}{di} = 24.07 \times 1352 = 32,543$.

The combination of the two bonds has present value $P = P_1 + P_2 = 11,352$, and the modified duration of the two bond portfolio is $-\frac{dP}{di} / P$. We first find

$$-\frac{dP}{di} = -\frac{dP_1}{di} + -\frac{dP_2}{di} = 98,200 + 32,543 = 130,743.$$

Then $-\frac{dP}{di} / P = 130,743 / 11,352 = 11.5$.