EXAM C QUESTIONS OF THE WEEK

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Week of October 9/06

Losses for the year for a risk are uniformly distributed on $(0, \theta)$, where θ is uniformly distributed on the interval $(1, 2)$.

The first year loss for a risk is L. Find the Buhlmann credibility premium for the second year's loss for the same risk in terms of L.

Solution can be found below.

Week of October 9/06 - Solution

Prior distribution is $\pi(\theta) = 1\,,\ 1 < \theta < 2$.

The hypothetical mean is $\mu(\theta) = E[X|\theta] = \frac{\theta}{2}$, since X is uniformly distributed on $(0, \theta)$.

Process variance is $v(\theta) = Var[X|\theta] = \frac{\theta^2}{12}$.

Expected hypothetical mean is $\mu = E[X] = E[\,E[X|\theta]\,] = E[\,\frac{\theta}{2}\,] = \frac{1.5}{2} = .75$.

Expected process variance is $v = E[\,Var[X|\theta]\,] = E[\,\frac{\theta^2}{12}\,] = \frac{1}{12} \cdot \int_1^2 \theta^2\,d\theta = \frac{7}{36}$.

Variance of the hypothetical mean is
$a = Var[\,E[X|\theta]\,] = Var[\,\frac{\theta}{2}\,] = \frac{1}{4}\,Var[\theta] = \frac{1}{4} \cdot \frac{1}{12} = \frac{1}{48}$.

There is one observation, so $Z = \frac{n}{n+\frac{v}{a}} = \frac{1}{1+\frac{7/36}{1/48}} = .0968$.

Buhlmann credibility premium is
$ZL + (1-Z)\mu = .0968L + .9032(.75) = .0968L + .6744$.