EXAM M QUESTIONS OF THE WEEK

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Question 11 - Week of October 3

An insurance policy on the loss X has an ordinary deductible of 40. The policy also has the following adjustments. If the loss is between 40 and 60, the insurance policy pays the amount of the loss above 40. If the loss is between 60 and 80, the insurance pays 20 plus 75% of the loss above 60. If the loss is above 80, the insurance pays 35.

(a) If the distribution of X is uniform on the interval $(0, 100)$, find the expected cost per loss .

(b) Express the cost per loss random variable as a combination of X and $X \wedge u$ factors for appropriate values of u.

The solution can be found below.

Question 11 Solution

(a) $Y_L = \begin{cases} 0 & X \leq 40 \\ X - 40 & 40 < X \leq 60 \\ 20 + .75(X - 60) & 60 < X \leq 80 \\ 35 & X > 80 \end{cases}$

$E[Y_L] = \int_{40}^{60} (x - 40)(.01)\, dx + \int_{60}^{80} [20 + .75(x - 60)](.01)\, dx + \int_{80}^{100} 35(.01)\, dx$
$= 2 + 5.5 + 7 = 14.5$.

(b) Amount paid by insurer (cost per loss) is

$$Y_L = \begin{cases} 0 & X \leq 40 \\ X - 40 & 40 < X \leq 60 \\ 20 + .75(X - 60) & 60 < X \leq 80 \\ 35 & X > 80 \end{cases}$$

$(X - 40)_+ = \begin{cases} 0 & X \leq 40 \\ X - 40 & X > 40 \end{cases}$ is valid for Y_L up to $X = 60$.

If we subtract $.25(X - 60)_+$, we get

$$(X - 40)_+ - .25(X - 60)_+$$
$$= \begin{cases} 0 & X \leq 40 \\ X - 40 & 40 < X \leq 60 \\ (X - 40) - .25(X - 60) = 20 + .75(X - 60) & X > 60 \end{cases}$$

If we subtract $.75(X - 80)_+$, we get

$$(X - 40)_+ - .25(X - 60)_+ - .75(X - 80)_+$$
$$= \begin{cases} 0 & X \leq 40 \\ X - 40 & 40 < X \leq 60 \\ 20 + .75(X - 60) & 60 < X \leq 80 \\ 20 + .75(X - 60) - .75(X - 80) = 35 & X > 80 \end{cases}$$

This is the cost per payment made by the insurance.

This is $(X - 40)_+ - .25(X - 60)_+ - .75(X - 80)_+$
$= X - (X \wedge 40) - .25[X - (X \wedge 60)] - .75[X - (X \wedge 80)]$
$= .75(X \wedge 80) + .25(X \wedge 60) - (X \wedge 40)$