EXAM M QUESTIONS OF THE WEEK

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Question 15 - Week of October 31

An auto insurance company has a rating system which rates drivers as Low, Medium, or High risk. A rating is assigned to the policyholder at the time an auto insurance policy is issued. The rating is updated at the end of each year. The transition probability matrix of risk rating in a policyholder's first year is

	L	M	H
L	.6	.3	.1
M	.3	.4	.3
H	.1	.2	.7

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For all years after the first year, the transition probability matrix of risk rating is

	L	M	H
L	.8	.1	.1
M	.2	.6	.2
H	0	.1	.9

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(a) Suppose that a new policyholder is rated as low risk. Find the probability that this policyholder will never be rated as high risk by the end of (and including) the third policy year.

(b) Based on a policyholder's rating at the start of a year, the insurance company estimates annual claims for that year as follows (assumed to be paid at the end of each year)
Low risk: \$100 ; Medium Risk: \$200 ; High Risk: \$1000 .
For a new policyholder rated Low risk, find the total expected claims for the first three years.

The solution can be found below.

Question 15 Solution

(a) We convert state H to an absorbing state. The transition matrices become

$$\boldsymbol{Q_0} = \begin{array}{c|ccc} & L & M & H \\ L & .6 & .3 & .1 \\ M & .3 & .4 & .3 \\ H & 0 & 0 & 1 \end{array}$$ for the first year,

and $$\boldsymbol{Q_n} = \begin{array}{c|ccc} & L & M & H \\ L & .8 & .1 & .1 \\ M & .2 & .6 & .2 \\ H & 0 & 0 & 1 \end{array}$$ for the second and subsequent years ($n \geq 1$).

Then $${}_{\boldsymbol{3}}\boldsymbol{Q} = \boldsymbol{Q_0} \times \boldsymbol{Q_1} \times \boldsymbol{Q_2} = \begin{array}{c|ccc} & L & M & H \\ L & .48 & .198 & .322 \\ M & .31 & .194 & .496 \\ H & 0 & 0 & 1 \end{array}$$

Then the probability in question is ${}_3Q_0^{(L,L)} + {}_3Q_0^{(L,M)} = .48 + .198 = .678$
(the probability of being rated L or M at the end of 3 years).

(b) For a policyholder rated Low risk at time 0, expected first year claim is 100.
The rating probabilities at the start of the second year for this policyholder are
L: $Q_0^{(L,L)} = .6$, M: $Q_0^{(L,M)} = .3$, H: $Q_0^{(L,H)} = .1$.
The expected claim in the 2nd year is $(.6)(100) + (.3)(200) + (.1)(1000) = 220$.

The two-step transition probabilities from time 0 are in the matrix

$${}_{\boldsymbol{2}}\boldsymbol{Q} = \begin{bmatrix} .6 & .3 & .1 \\ - & - & - \\ - & - & - \end{bmatrix} \times \begin{bmatrix} .8 & .1 & .1 \\ .2 & .6 & .2 \\ 0 & .1 & .9 \end{bmatrix} = \begin{bmatrix} .54 & .25 & .21 \\ - & - & - \\ - & - & - \end{bmatrix}.$$

The rating probabilities at the start of the third year for this policyholder are
L: ${}_2Q_0^{(L,L)} = .54$, M: $2Q_0^{(L,M)} = .25$, H: $2Q_0^{(L,H)} = .21$.
The expected claim in the 3rd year is $(.54)(100) + (.25)(200) + (.21)(1000) = 314$.
Three-year total expected claim is 634 .