

EXAM M QUESTIONS OF THE WEEK

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Week of October 30/06

A retirement residence has the facilities to accommodate individuals who can live independently, require assisted living or have a temporary disability. The facility models the residents using a multi-state transition model approach with four states:

I- living independently, *A*-requiring assisted living accommodation,

D-living in accommodation for those with a disability, and *O*-out of the facility.

A resident's state is updated on Jan. 1 of each year.

The one-step transition probability matrices for Jan. 1, 2006 to Jan. 1, 2007 and Jan. 1, 2007 to

Jan. 1, 2008 are
$$\begin{bmatrix} .7 & .1 & .1 & .1 \\ .5 & .2 & .2 & .1 \\ .2 & .2 & .4 & .2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
 and
$$\begin{bmatrix} .8 & .1 & .05 & .05 \\ .6 & .25 & .1 & .05 \\ .3 & .25 & .3 & .15 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(the states for the matrices are in the order *I*, *A*, *D*, *O* from top to bottom and left to right).

Annual health care costs for a resident is 300 for someone living independently, 700 for someone in assisted living, and 2000 for someone who is disabled. On January 1, 2006 there are 200 residents in the facility, of whom 150 are living independently, 40 are in assisted living and 10 are disabled. Find the expected health care costs for these residents for years 2006, 2007 and 2008 combined.

The solution can be found below.

Week of October 30/06 - Solution

The 2-step transition probability matrix from Jan. 1, 2006 to Jan.1, 2008 is

$$\begin{bmatrix} .7 & .1 & .1 & .1 \\ .5 & .2 & .2 & .1 \\ .2 & .2 & .4 & .2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} .8 & .1 & .05 & .05 \\ .6 & .25 & .1 & .05 \\ .3 & .25 & .3 & .15 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} .65 & .12 & .075 & .155 \\ .58 & .15 & .105 & .165 \\ .4 & .17 & .15 & .28 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Expected health care costs for someone living independently on Jan. 1, 2006 will be 300 in 2006, $(.7)(300) + (.1)(700) + (.1)(2000) = 480$ in 2007, and $(.65)(300) + (.12)(700) + (.075)(2000) = 429$ in 2008, for a total of 1209 .

Expected health care costs for someone living with assistance on Jan. 1, 2006 will be 700 in 2006, $(.5)(300) + (.2)(700) + (.2)(2000) = 690$ in 2007, and $(.58)(300) + (.15)(700) + (.105)(2000) = 489$ in 2008, for a total of 1879 .

Expected health care costs for someone disabled on Jan. 1, 2006 will be 2000 in 2006, $(.2)(300) + (.2)(700) + (.4)(2000) = 1000$ in 2007, and $(.4)(300) + (.17)(700) + (.15)(2000) = 539$ in 2008, for a total of 3539 .

Total expected costs for all residents are

$$(150 \times 1209) + (40)(1879) + (10)(3539) = 291,900 .$$