EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 14 - Week of October 24

A Poisson process $\{N(t) : t \geq 0\}$ has a rate of $\lambda = 3$ per unit time.
Events are classified as Type 1 and Type 2.
When an event occurs, there is a $\frac{1}{3}$ probability that it is a Type 1 event and a $\frac{2}{3}$ probability that it is a Type 2 event. Event types are independent of one another. If an answer involves exponential factors, leave it in exponential form.

(a) Find the probability that the 2nd event of Type 1 occurs before the 3rd event of Type 2.

(b) $\{N_1(t) : t \geq 0\}$ is the process of Type 1 events.
Find each of the following
(i) $E[N_1(1)|N(1) = 3]$, (ii) $E[N(1)|N_1(1) = 1]$, (iii) $Cov[N(1), N_1(1)]$

The solution can be found below.

Question 14 Solution

(a) $P[S_2^{(1)} < S_3^{(2)}] = P[\text{at least 2 of the first 4 events are Type 1}]$
$= \binom{4}{2}(\frac{1}{3})^2(\frac{2}{3})^2 + \binom{4}{3}(\frac{1}{3})^3(\frac{2}{3})^1 + \binom{4}{4}(\frac{1}{3})^4(\frac{2}{3})^0$
$= 6(\frac{1}{9})(\frac{4}{9}) + 4(\frac{1}{27})(\frac{2}{3}) + \frac{1}{81} = \frac{33}{81} = \frac{11}{27}$.

(b)(i) Given that $N(1) = 3$, the number of events of Type 1 has a binomial distribution $n = 3$, $p = \frac{1}{3}$, so the expected number of Type 1 events is 1.

(ii) $N(1) = N_1(1) + N_2(1) \rightarrow E[N(1)|N_1(1) = 1] = 1 + E[N_2(1)|N_1(1)]$
$= 1 + E[N_2(1)] = 1 + 2 = 3$ since $N_1(1)$ and $N_2(1)$ are independent, and $N_2(1)$ is a Poisson process with rate $\lambda(\frac{2}{3}) = 2$.

(iii) $Cov[N(1), N_1(1)] = Cov[N_1(1) + N_2(1), N_1(1)]$
$= Cov[N_1(1), N_1(1)] + Cov[N_2(1), N_1(1)] = Var[N_1(1)] + 0$ (independence) $= 1$.