

EXAM FM QUESTIONS OF THE WEEK

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Question 14 - Week of October 24

Smith takes out a mortgage on 1/1/94 with the following provisions:

Amount of mortgage: \$100,000

Interest rate: 12% per year, compounded monthly

Number of repayments: 360

Frequency of repayments: Monthly

Date of first repayment: 1/31/94

On 12/31/98, after making the 12/31/98 repayment, Smith repays an additional \$3,000 to reduce the outstanding balance of his mortgage. He will continue to make the same monthly repayments he has been making (of the outstanding balance of the mortgage, if less) until the outstanding balance of his mortgage is reduced to zero.

In what range is the total interest paid by Smith?

The solution can be found below.

Question 14 Solution

The monthly payment is $\frac{100,000}{a_{\overline{360}|.01}} = 1,028.61$. The payment on 12/31/98 is the 60th payment, so the outstanding balance just after that payment is $1,028.61 \cdot a_{\overline{300}|.01} = 97,663$.

After the additional payment of 3,000 the balance is reduced to 94,663. The number of payments of 1,028.61 needed to repay the loan is n , where $94,663 = 1,028.61 \cdot a_{\overline{n}|.01}$.

Using the unknown time function of a calculator (or solving directly for n from the equation $94,663 = 1,028.61 \cdot \left(\frac{1-v_{.01}^n}{.01}\right)$) we get $n = 254.2$. Therefore, there will be 254 more full payments of 1,028.61 and a final 255-th payment of amount approximately $.2 \times 1,028.61 = 206$. The total amount paid over the course of the loan is

$$1,028.61 \times 60 + 3,000 + 1,028.61 \times 254 + 206 = 326,190.$$

The total amount of interest paid over the course of the loan is $326,190 - 100,000 = 226,190$.

Note that we could have found the exact amount X of the smaller 255-th payment from $94,663 = 1,028.61 \cdot a_{\overline{254}|.01} + Xv_{.01}^{255}$ ($X = 218$), but the difference from the approximate value of 206 is insignificant.