

## EXAM C QUESTIONS OF THE WEEK

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### Question 14 - Week of October 24

You are given the following 3 data points:

$$\begin{array}{rcccc} x : & 0 & 1 & 2 \\ y : & 1 & 2 & 0 \end{array}$$

Find the natural cubic spline for the data points.

Find  $f'_0(0)$  and  $f'_1(2)$  for this spline.

The solution can be found below.

### **Question 14 Solution**

Use  $h_0m_0 + 2(h_0 + h_1)m_1 + h_1m_2 = 6\left[\frac{y_2 - y_1}{h_1} - \frac{y_1 - y_0}{h_0}\right]$  and  
 $a_j = y_j$ ,  $b_j = \frac{y_{j+1} - y_j}{h_j} - \frac{h_j(2m_j + m_{j+1})}{6}$ ,  $c_j = \frac{m_j}{2}$  and  $d_j = \frac{m_{j+1} - m_j}{6h_j}$ .

For the natural spline  $m_0 = m_2 = 0 \rightarrow 2(1 + 1)m_1 = 6\left[\frac{0-2}{1} - \frac{2-1}{1}\right] \rightarrow m_1 = -\frac{9}{2}$ .

Then  $a_0 = y_0 = 1$ ,  $b_0 = \frac{y_1 - y_0}{h_0} - \frac{h_0(2m_0 + m_1)}{6} = 1 - \frac{(-9/2)}{6} = \frac{7}{4}$ ,  $c_0 = \frac{m_0}{2} = 0$  and

$d_0 = \frac{m_1 - m_0}{6h_0} = \frac{-9/2}{6} = -\frac{3}{4}$ , and

$a_1 = y_1 = 2$ ,  $b_1 = \frac{y_2 - y_1}{h_1} - \frac{h_1(2m_1 + m_2)}{6} = \frac{0-2}{1} - \frac{(-9)}{6} = -\frac{1}{2}$ ,  $c_1 = \frac{m_1}{2} = -\frac{9}{4}$  and

$d_1 = \frac{m_2 - m_1}{6h_1} = \frac{9/2}{6} = \frac{3}{4}$ .

Then  $f_0(x) = 1 + \frac{7}{4}x - \frac{3}{4}x^3$  and  $f_1(x) = 2 - \frac{1}{2}(x - 1) - \frac{9}{4}(x - 1)^2 + \frac{3}{4}(x - 1)^3$ , and

$f'_0(x) = \frac{7}{4} - \frac{9}{4}x^2$  so that  $f'_0(0) = \frac{7}{4}$

and  $f'_1(x) = -\frac{1}{2} - \frac{9}{2}(x - 1) + \frac{9}{4}(x - 1)^2$  so that  $f'_1(2) = -\frac{1}{2} - \frac{9}{2} + \frac{9}{4} = -\frac{11}{4}$ .