EXAM M QUESTIONS OF THE WEEK

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Week of October 23/06

24-hour Lube-n-Tune auto shop does oil changes and tune-ups for automobiles.
Cars arrive at Lube-n-Tune according to a Poisson process at a rate of 4 per hour (during business hours): 50% of cars arriving are for an oil change only, 25% are for a tune-up only, and 25% are for an oil change and tune-up.

Find the probability that between 10AM and 11 AM at least two cars arrive requesting an oil change and at least one car arrives requesting a tune up.

The solution can be found below.

Week of October 23/06 - Solution

We have the following independent sub-processes:
- cars arrive requesting an oil change only at a rate of 2 per hour,
- cars arrive requesting a tune-up only at a rate of 1 per hour,
- cars arrive requesting an oil change and tune-up at a rate of 1 per hour.

A is the event of 0 or 1 cars arriving requesting an oil change between 10 and 11,
B is the event of 0 cars arriving requesting a tune up between 10 and 11.

The probability in question is $P(A' \cap B') = 1 - P(A \cup B)$.

The number of cars arriving between 10 and 11 requesting an oil change is Poisson with a mean of $2 + 1 = 3$, and the number of cars arriving between 10 and 11 requesting a tune-up is Poisson with a mean of $1 + 1 = 2$.
$P(A) = e^{-3}[1+3] = 4e^{-3}$, $P(B) = e^{-2}$.
$P(A \cap B) = P($no cars arriving between 10 and 11$)$
$\quad + P($1 car arriving between 10 and 11 requesting an oil change
$\qquad \cap$ no cars arriving between 10 and 11 requesting a tune-up$)$.
$P($no cars arriving between 10 and 11$) = e^{-4}$, and

$P($1 car arriving between 10 and 11 requesting an oil change
$\qquad \cap$ no cars arriving between 10 and 11 requesting a tune-up$)$
$= P($1 car arriving between 10 and 11 requesting an oil change <u>only</u>
$\qquad \cap$ no cars arriving between 10 and 11 requesting a tune-up$)$
$= (2e^{-2})(e^{-2}) = 2e^{-4}$ (the two events in the intersection are independent).

Then $P(A \cup B) = 4e^{-3} + e^{-2} - 3e^{-4}$, and the probability in question is
$1 - (4e^{-3} + e^{-2} - 3e^{-4}) = .72$.

Thanks for "ctperng" posting the following solution on the Actuary.Com Discussion Forum.
The probability can be formulated as
$P(\text{oil + tune} \geq 2) + P(\text{oil + tune} = 1) \times P(\text{oil only} \geq 1)$
$+ P(\text{oil + tune} = 0) \times P(\text{oil only} \geq 2) \times P(\text{tune only} \geq 1)$
$= (1 - e^{-2} - 2e^{-2}) + (e^{-1})(1 - e^{-2}) + (e^{-1})(1 - e^{-2} - 2e^{-2})(1 - e^{-1})$
$= 1 - (4e^{-3} + e^{-2} - 3e^{-4}) = .72$.