

EXAM C QUESTIONS OF THE WEEK

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Week of October 23/06

X is the random variable denoting the number of claims in one day. The following is a sample of the number of claims occurring on 5 randomly chosen days:

2 0 4 5 7

The following estimator from a sample of n days is used to estimate $P(X \leq 4)$, the probability of 4 or less claims in one day: $\hat{\theta} = \frac{\# \text{ days with 4 claims or less}}{n}$

The bootstrap approximation is applied to estimate the mean square error of $\hat{\theta}$ using the following 8 samples simulated from the empirical distribution of the original sample:

Sample 1	0	2	0	7	7
Sample 2	2	2	4	2	5
Sample 3	7	2	5	4	5
Sample 4	4	0	7	2	5
Sample 5	2	7	2	5	5
Sample 6	4	5	4	2	7
Sample 7	0	0	2	4	7
Sample 8	5	0	4	7	5

Find the bootstrap approximation.

Solution can be found below.

Week of October 23/06 - Solution

In the empirical distribution, the actual value of the parameter being estimated is $\theta = \frac{3}{5} = .6$, since three of the values in the 5-point empirical distribution are ≤ 4 .

We calculate $\hat{\theta}$ and $(\hat{\theta} - \theta)^2 = (\hat{\theta} - .6)^2$ for each simulated sample.

						$\hat{\theta}$	$\theta = .6$	$(\hat{\theta} - .6)^2$
Sample 1	0	2	0	7	7	.6		0
Sample 2	2	2	4	2	5	.8		.04
Sample 3	7	2	5	4	5	.4		.04
Sample 4	4	0	7	2	5	.6		0
Sample 5	2	7	2	5	5	.4		.04
Sample 6	4	5	4	2	7	.6		0
Sample 7	0	0	2	4	7	.8		.04
Sample 8	5	0	4	7	5	.2		.04

The average value of $(\hat{\theta} - .6)^2$ is $\frac{5 \times .04}{8} = .025$; this is the bootstrap approximation to the mean square error of $\hat{\theta}$ based on the 8 samples.