EXAM M QUESTIONS OF THE WEEK

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Question 13 - Week of October 17

A compound Poisson claim distribution S has Poisson parameter $\lambda = 2$ and severity distribution $X = \begin{cases} 1 & \text{prob. } .4 \\ 2 & \text{prob. } .2 \\ 3 & \text{prob. } .4 \end{cases}$

(a) Find $E[S]$ and $Var[S]$.

(b) Find $P[S = 3]$ two ways:
(i) using a combinatorial approach, and
(ii) using the recursive method.

(c) A deductible of 1 is applied to each individual claim X. The aggregate insurance payment (after deductibles are applied) is S'. Describe the modified frequency and severity, N' and X'. Find $E[S'] = E[N'] \cdot E[X']$ and $E[S'] = E[N] \cdot E[(X-1)_+]$.

(d) Stop loss insurance with a deductible of 2 is applied to S. Find $E[(S-2)_+]$.

The solution can be found below.

Question 13 Solution

(a) $E[S] = 2[(.4 + .4 + 1.2] = 4$, $Var[S] = 2[.4 + .8 + 3.6] = 9.6$.

(b)(i) $P[S = 3] = P[N = 1] \cdot P[X = 3] + P[N = 2] \cdot 2 \cdot P[X = 1] \cdot P[X = 2]$
$+ P[N = 3] \cdot (P[X = 1])^3$
$= \frac{e^{-2}2}{1}(.4) + 2 \cdot \frac{e^{-2}2^2}{2}(.4)(.2) + \frac{e^{-2}2^3}{6}(.4)^3 = 1.2053e^{-2} = .163$

(ii) $P[S = k] = g_k = \frac{\lambda}{k}\sum_{j=1}^{k} j \cdot f_j \cdot g_{k-j}$, where $f_j = P[X = j]$
$g_0 = P[S = 0] = P[N = 0] = e^{-2}$
$P[S = 1] = g_1 = 2[f_1 g_0] = 2(.4)(e^{-2}) = .8e^{-2}$
$P[S = 2] = g_2 = \frac{2}{2}[f_1 g_1 + 2f_2 g_0] = (.4)(.8e^{-2}) + 2(.2)(e^{-2}) = .72e^{-2}$
$P[S = 3] = g_3 = \frac{2}{3}[f_1 g_2 + 2f_2 g_1 + 3f_3 g_0] = \frac{2}{3}[(.4)(.72e^{-2}) + 2(.2)(.8e^{-2}) + 3(.4)e^{-2}]$
$= 1.2053e^{-2}$

(c) $P[X > 1] = .6$, N' has a Poisson distribution with mean $(2)(.6) = 1.2$
$X' =$ Conditional dist of $X - 1|X > 1 = \begin{cases} 1 & \text{prob. } 1/3 \\ 2 & \text{prob. } 2/3 \end{cases}$.
$E[N']E[X'] = (1.2)(5/3) = 2$
$E[(X - 1)_+] = 1(.2) + 2(.4) = 1 \rightarrow E[N]E[(X - 1)_+] = (2)(1) = 2$

(d) $E[(S - 2)_+] = E[S] - [1 - F(0)] - [1 - F(1)] = 4 - (1 - f(0)) - (1 - f(0) - f(1))$
$= 4 - (1 - e^{-2}) - (1 - e^{-2} - .8e^{-2}) = 2 + 2.8e^{-2}$.