EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 12 - Week of October 10

A continuous loss random variable X is uniformly distributed on the interval $(0, 100)$.

(a) If a loss is over 20 but less than 80, an insurance policy pays 50% of the loss amount that is over 20. If a loss is over 80, the insurance pays the full loss amount over 20. Find the expected cost per payment for this insurance.

(b) If $X < 50$, a risk manager is paid a bonus equal to 50% of the difference between X and 50.
(i) Find the mean and variance of the bonus received by the risk manager.
(ii) Find the expected bonus, given that $X < 50$.

The solution can be found below.

Question 12 Solution

(a) $\int_{20}^{80} .5(x-20)(.01)\,dx + \int_{80}^{100}(x-20)(.01)\,dx = 9+14 = 23$ is expected cost per loss.
Expected cost per payment is $\frac{23}{P[X>20]} = \frac{23}{.8} = 28.75$.

(b)(i) Bonus $= .5 \times \begin{cases} 50-X & X<50 \\ 0 & X>50 \end{cases} = .5[50-(X \wedge 50)]$

$E[\text{Bonus}] = .5[50 - E[X \wedge 50]] = .5[50 - [\int_0^{50} x(.01)\,dx + 50P[X>50]]]$
$= .5[50-[12.5+25]] = 6.25$ or
$E[\text{Bonus}] = .5 \times \int_0^{50}(50-x)(.01)\,dx = 6.25$

$E[\text{Bonus}^2] = (.5)^2 \times \int_0^{50}(50-x)^2(.01)\,dx = 104.17$

Variance of bonus $= 104.17 - (6.25)^2 = 65.1$.

(ii) $E[\text{Bonus}|X<50] = \frac{E[Bonus]}{P[X<50]} = 12.5$.