EXAM C QUESTIONS OF THE WEEK

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Question 12 - Week of October 10

You are given the following table of data for two policyholders over a three year period.

Policy Year →	1	2	3
Policyholder ↓	Claim Amount		
1	25	40	55
2	55	65	60

Apply non-parametric empirical Bayesian analysis to estimate μ, v, a and Z, and find the estimated credibility premium for Policyholder 1 for the 4th year.

The solution can be found below.

Question 12 Solution

$r = 2$ policyholders (groups) and $n_1 = n_2 = n_3 = 3 = m_1 = m_2$ exposure periods (years) for each group, and $m_{ij} = 1$ exposure unit for combination of group and year.

$\overline{X}_1 = \frac{25+40+55}{3} = 40$, $\overline{X}_2 = \frac{55+65+60}{3} = 60$, $\overline{X} = \frac{40+60}{2} = 50 = \widehat{\mu}$,

$\widehat{v} = \frac{1}{r(n-1)} \sum_{i=1}^{r} \sum_{j=1}^{n} (X_{ij} - \overline{X}_i)^2$

$= \frac{1}{(2)(2)} \left([(25-40)^2 + (40-40)^2 + (55-40)^2] + [(55-60)^2 + (65-60)^2 + (60-60)^2] \right)$

$= 125.0$, and

$\widehat{a} = \frac{1}{r-1} \sum_{i=1}^{r} (\overline{X}_i - \overline{X})^2 - \frac{\widehat{v}}{n} = \frac{1}{1}[(40-50)^2 + (60-50)^2] - \frac{125}{3} = \frac{475}{3}$.

Then $\widehat{k} = \frac{\widehat{v}}{\widehat{a}} = \frac{125}{475/3} = .7895$, and the estimated credibility factor for group 1 is

$\widehat{Z}_1 = \frac{m_1}{m_1 + \widehat{k}} = \frac{3}{3+.7895} = .7917$. The credibility premium for group 1 for the fourth year is

$\widehat{Z}_1 \overline{X}_1 + (1 - \widehat{Z}_1)\widehat{\mu} = (.7917)(40) + (.2083)(50) = 42.08$.