EXAM MLC QUESTIONS OF THE WEEK

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Week of September 3/07

A special fully discrete 20-year endowment insurance policy issued to (x) has a death benefit of $1000 + {}_kV$ if (x) dies in the k-th year for $k = 1, 2, ..., 20$. If (x) survives to age $x + 20$, the endowment benefit is 1000. The annual effective rate of interest is 8% and mortality probability is $q_y = .02$ for $y = x, x + 1, ...$. Find the variance of the issue date loss random variable.

The solution can be found below.

Week of September 3/07 - Solution

We use the recursive variance relationship

$Var[{}_hL|K(x) \geq h] = [v(b_{h+1} - {}_{h+1}V)]^2\, p_{x+h}\, q_{x+h} + v^2 p_{x+h} Var[{}_{h+1}L|K(x) \geq h+1]$.

For $h = 19$ we have
$Var[{}_{19}L|K(x) \geq 19] = [v(b_{20} - {}_{20}V)]^2\, p_{x+19}\, q_{x+19} + v^2 p_{x+19} Var[{}_{20}L|K(x) \geq 20]$.

For this 20-year policy, ${}_{20}L =$ endowment amount $= 1000$, so $Var[{}_{20}L|K(x) \geq 20] = 0$.
Therefore, since $b_{20} = 1000 + {}_{20}V$ it follows that
$Var[{}_{19}L|K(x) \geq 19] = [v(b_{20} - {}_{20}V)]^2\, p_{x+19}\, q_{x+19} = (1000v])^2(.98)(.02) = 16,804$.

Then, since $b_{19} = 1000 + {}_{19}V$, we have
$Var[{}_{18}L|K(x) \geq 18] = [v(b_{19} - {}_{19}V)]^2\, p_{x+18}\, q_{x+18} + v^2 p_{x+18} Var[{}_{19}L|K(x) \geq 19]$.
$= (1000v])^2(.98)(.02) + .98v^2(16,804) = 16,804 + .840192(16,804)$.

Then, since $b_{18} = 1000 + {}_{18}V$, we have
$Var[{}_{17}L|K(x) \geq 17] = [v(b_{18} - {}_{18}V)]^2\, p_{x+17}\, q_{x+17} + v^2 p_{x+17} Var[{}_{18}L|K(x) \geq 18]$.
$= (1000v])^2(.98)(.02) + .840192[16,804 + .840192(16,804)]$
$= 16,804[1 + c + c^2]$, where $c = .840192$.

Continuing in this way, we see that
$Var[{}_0L] = 16,804[1 + c + c^2 + \cdots + c^{19}] = 16,804 \cdot \frac{1-.840192^{20}}{1-.840192} = 101,920$.