

EXAM MLC QUESTIONS OF THE WEEK

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Week of September 17/07

For a population comprised of smokers and nonsmokers, you are given:

- (i) Mortality of nonsmokers is based on DeMoivre's Law with $\omega = 90$.
- (ii) The force of mortality for smokers is β times as large as that for nonsmokers at all ages.

For a pair of individuals with independent lifetimes, each at age (40), one a nonsmoker and one a smoker, the complete expected time until the second death is 30.71 years. Find β .

- A) Less than 1 B) At least 1, but less than 1.2 C) At least 1.2, but less than 1.4
D) At least 1.4, but less than 1.6 D) At least 1.6

The solution can be found below.

Week of September 17/07 - Solution

Nonsmokers have force of mortality $\mu^N(y) = \frac{1}{90-y}$, and smokers have force of mortality $\mu^S(y) = \frac{\beta}{90-y}$. Under DeMoivre's Law, survival probability is ${}_t p_x^N = \frac{\omega-x-t}{\omega-x}$ and force of mortality is $\mu^N(y) = \frac{1}{\omega-y}$.

Since smokers have force of mortality $\mu^S(y) = \frac{\beta}{\omega-y}$, it follows that smoker survival is ${}_t p_x^S = \left(\frac{\omega-x-t}{\omega-x}\right)^\beta$, with $0 < t < \omega - x$. Both nonsmokers and smokers have upper age limit 90.

Last survivor expectation for two independent lives aged 40, one a nonsmoker and one a smoker

$$\begin{aligned} \text{is } \ddot{e}_{40^N:40^S} &= \ddot{e}_{40^N} + \ddot{e}_{40^S} - \ddot{e}_{40^N:40^S} \\ &= \int_0^{50} \left(\frac{50-t}{50}\right) dt + \int_0^{50} \left(\frac{50-t}{50}\right)^\beta dt - \int_0^{50} \left(\frac{50-t}{50}\right) \left(\frac{50-t}{50}\right)^\beta dt \\ &= \frac{50}{2} + \frac{50}{\beta+1} - \frac{50}{\beta+2} = 30.7143. \end{aligned}$$

From this equation, we get $\frac{1}{\beta+1} - \frac{1}{\beta+2} = .114286$. This results in the quadratic equation $\beta^2 + 3\beta - 6.75 = 0$. The two roots are $\beta = 1.5$ and $\beta = -4.5$.

We ignore the negative root.