

# EXAM MLC QUESTION OF THE WEEK

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## Week of May 12/08

Jojo McGoon is the team tough guy for the Bay City Brawlers hockey team.

The number of penalties Jojo gets in a game follows a Poisson process with a rate of 6 penalties per game on average. Jojo gets three types of penalties, and they occur independently of one another.

The penalty types and probabilities are:

Type 1 - minor penalty,  $1/2$  of all penalties are minor, on average

Type 2 - major penalty,  $1/3$  of all penalties are major, on average

Type 3 - serious penalty,  $1/6$  of all penalties are serious, on average.

The hockey league fines the Brawlers \$1000 for each major penalty and \$2000 for each serious penalty, but there is no fine for a minor penalty.

What is the expected time (in minutes) that Jojo's fines first reach \$2000? Assume that a game is 60 minutes in length and also assume that if Jojo has no penalties in a game, the time count continues with the start of the next game.

**The solution can be found below.**

## Week of May 12/08 - Solution

Let  $T$  denote the time (in games) that total fines first reach 2000. Then

$$P(T > t) = P(\text{at most one major penalty and no serious penalties by time } t) \\ = P[N_2(t) = 0, 1 \cap N_3(t) = 0] = [e^{-2t} + 2te^{-2t}] \times e^{-t} = e^{-3t} + 2te^{-3t} .$$

$$\text{Then, } E(T) = \int_0^{\infty} P(T > t) dt = \int_0^{\infty} [e^{-3t} + 2te^{-3t}] dt \\ = \frac{1}{3} + \frac{2}{9} = \frac{5}{9} \text{ games, or } 33.3 \text{ minutes.}$$