EXAM MLC QUESTION OF THE WEEK

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Week of March 17/08

A 2 decrement model beginning at age (x) has the following forces of decrement:

$\mu_x^{(1)}(t) = \frac{1}{40-t}$ $0 < t < 40$ and $\mu_x^{(2)}(t) = \frac{2}{80-t}$ $0 < t < 80$

Find ${}_{5|5}q_x^{(1)}$.

The solution can be found below.

Week of March 17/08 - Solution

$_{5|5}q_x^{(1)} = {}_{10}q_x^{(1)} - {}_5q_x^{(1)} = \int_0^{10} {}_tp_x^{(\tau)} \cdot \mu_x^{(1)}(t)\,dt - \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_x^{(1)}(t)\,dt$.

${}_tp_x^{(\tau)} = exp[-\int_0^t (\mu_x^{(1)}(s) + \mu_x^{(2)}(s))\,dt = exp[-\int_0^t (\frac{1}{40-s} + \frac{2}{80-s})\,dt$

$= exp[-(ln(\frac{40}{40-t}) + 2\,ln(\frac{80}{80-t})] = (\frac{40-t}{40})(\frac{80-t}{80})^2$.

$\int_0^{10} {}_tp_x^{(\tau)} \cdot \mu_x^{(1)}(t)\,dt = \int_0^{10} (\frac{40-t}{40})(\frac{80-t}{80})^2(\frac{1}{40-t})\,dt = \frac{1}{40\times 80^2}\int_0^{10}(80-t)^2\,dt = .220052$.

$\int_0^5 {}_tp_x^{(\tau)} \cdot \mu_x^{(1)}(t)\,dt = \int_0^5 (\frac{40-t}{40})(\frac{80-t}{80})^2(\frac{1}{40-t})\,dt = \frac{1}{40\times 80^2}\int_0^5(80-t)^2\,dt = .117350$.

$_{5|5}q_x^{(1)} = .220052 - .117350 = .103$.