EXAM MLC QUESTIONS OF THE WEEK

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Week of March 12/07

(SOA) For a special fully continuous last-survivor whole life insurance of 1 on (x) and (y), you are given:

(i) The premium is payable until the first death.

(ii) The independent random variables $T^*(x)$, $T^*(y)$, and Z are the components of a common shock model.

(iii) $T^*(x)$ has an exponential distribution with mean 25.

(iv) $T^*(y)$ has an exponential distribution with mean 16.67.

(v) Z, the common shock random variable, has an exponential distribution with mean 50.

(vi) $\delta = 0.04$

Calculate the annual benefit premium.

The solution can be found below.

Week of March 12/07 - Solution

If premium is at rate P per year payable continuously, then APV of premium is $P\overline{a}_{xy} = P\int_0^{\infty} e^{-\delta t}\,{}_tp_{xy}\,dt$. Under the common shock model, ${}_tp_{xy} = s_{T^*(x)}(t)\cdot s_{T^*(y)}(t)\cdot e^{-\lambda t}$, where λ is the factor from exponential distribution of the common shock. In this case, the mean of the common shock exponential distribution is $50 = \frac{1}{\lambda}$, so that $\lambda = .02$. Furthermore, since $T^*(x)$ has an exponential distribution with mean 25, the survival function for $T^*(x)$ must be $s_{T^*(x)}(t) = e^{-\frac{1}{25}t} = e^{-.04t}$, and similarly, for $T^*(y)$, $s_{T^*(y)}(t) = e^{-\frac{1}{16.67}t} = e^{-.06t}$.

This is the same as saying that $T^*(x)$ has constant force $\mu_x^* = .04$ and $T^*(y)$ has constant force $\mu_y^* = .06$. Then ${}_tp_x = e^{-t(.04+.02)} = e^{-.06t}$ so $T(x)$ has constant force $\mu_x = .06$, ${}_tp_y = e^{-t(.06+.02)} = e^{-.08t}$ so $T(y)$ has constant force $\mu_y = .08$, and ${}_tp_{xy} = s_{T^*(x)}(t)\cdot s_{T^*(y)}(t)\cdot e^{-\lambda t} = e^{-t(.04+.06+.02)} = e^{-.12t}$ so $T(xy)$ has constant force $\mu_{xy} = .12$. Then $\overline{a}_{xy} = \frac{1}{\mu_{xy}+\delta} = \frac{1}{.12+.04}$, so that $P\overline{a}_{xy} = \frac{1}{.16}\cdot P = 6.25P$.

The APV of the benefit is

$$\overline{A}_{\overline{xy}} = \overline{A}_x + \overline{A}_y - \overline{A}_{xy} = \frac{\mu_x}{\mu_x+\delta} + \frac{\mu_y}{\mu_y+\delta} - \frac{\mu_{xy}}{\mu_{xy}+\delta}$$

$$= \frac{.06}{.06+.04} + \frac{.08}{.08+.04} - \frac{.12}{.12+.04} = .6 + .667 - .75 \rightarrow P = .083 .$$