

EXAM MLC QUESTIONS OF THE WEEK

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Week of January 29/07

Z_1 denotes the present value random variable for a policy issued to (x) which has a benefit of 1 payable at the end of 20 years if (x) dies within 20 years. Z_2 is the present value random variable for a policy issued to (x) with a benefit of 1 payable at the end of 30 years if (x) dies between 10 and 30 years from the issue date. You are given

(i) $\text{Cov}[Z_1, Z_2] = 0$, (ii) ${}_{10}q_x = .16$ and (iii) ${}_{20}q_x = .46$. What is ${}_{30}q_x$?

A) .68 B) .72 C) .77 D) .81 E) .86

The solution can be found below.

Week of January 29/07 - Solution

$$Z_1 = \begin{cases} v^{20} & \text{if } T < 20 \\ 0 & \text{if } T \geq 20 \end{cases}, Z_2 = \begin{cases} 0 & \text{if } T < 10 \\ v^{30} & \text{if } 10 \leq T < 30 \\ 0 & \text{if } T \geq 30 \end{cases}$$

$$\rightarrow Z_1 \cdot Z_2 = \begin{cases} 0 & \text{if } T < 10 \\ v^{50} & \text{if } 10 \leq T < 20 \\ 0 & \text{if } T \geq 20 \end{cases}.$$

$$E[Z_1] = v^{20} \cdot {}_{20}q_x, \quad E[Z_2] = v^{30} \cdot {}_{10|20}q_x, \quad E[Z_1 \cdot Z_2] = v^{50} \cdot {}_{10|10}q_x$$

$$\rightarrow \text{Cov}[Z_1, Z_2] = E[Z_1 \cdot Z_2] - E[Z_1] \cdot E[Z_2] = v^{50} \cdot ({}_{10|10}q_x - {}_{20}q_x \cdot {}_{10|20}q_x) = 0$$

$$\rightarrow {}_{20}q_x - {}_{10}q_x - {}_{20}q_x \cdot ({}_{30}q_x - {}_{10}q_x) = 0 \rightarrow {}_{30}q_x = \frac{.46 - .16}{.46} + .16 = .81.$$